Beamforming Design and Power Control for Spectrum Sharing Systems

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Abstract

In order to provide wireless services for the current demand of high data rate mobile applications, more spectrally efficient systems are needed. As a matter of fact, the current wireless systems are limited by a frequency splitting spectrum management which on one hand minimizes the multiuser interference but; on the other hand, it precludes the use of wider bandwidth signals. As a more aggressive frequency reuse is targeted (ideally, all transmitters might eventually share the same frequency band), the use of multiple antennas for interference reliving, jointly with a smart power allocation is compulsory. In addition, novel spectrum management regulatory policies are required for ensuring a peaceful coexistence between adjacent spectrum sharing networks and for promoting their development.

The aim of this dissertation is provide a beamforming and power allocation design for these novel spectrum sharing systems which are meant to exponentially increase the spectral efficiency of the systems. A mathematical framework based on multi-criteria optimization for analyzing the beamforming design is provided which serves as a fundamental tool for describing the state-of-the-art studies in multiantenna interference networks. Indeed, the achievable rates are described and several ways of computing the Pareto rate region of MISO interference channel (i.e. the communication model that represents the spectrum sharing network when the transmitters use multiple antennas) are studied. Nevertheless, as the system designer aims to work in a single efficient rate point, the sum-rate optimal beamforming design is studied. Curiously, it results that under some realistic assumptions on both the desired and interference power levels, the obtained beamformer is the reciprocal version of a known receiving one and it optimizes a notion of antenna directivity for multiuser communications.

Nevertheless, it is important to remark that the higher transmit power is used, the more interference dominated is the medium, not only within the wireless network, but also to eventually adjacent networks that might suffer from inter-network interference. In order to cope with this problem, a spectrum licensing system is revisited, namely time-area-spectrum license. Under this spectrum management mechanism, a license holder is able to radiate signals under a certain portion of time, within a concrete area and in a given band. Moreover, the amount of signal strength within the area is constraint by a certain value. Since controlling the signal power levels in a given area is cumbersome, we propose to restrict the receive power as an estimation of the overall accumulated signal strength. Therefore, the optimal transmit beamformers and power allocations are studied. Concretely, the achievable rates are derived and an operational working point is envisaged. In addition, a sub-optimal yet low computationally complex and decentralized beamforming design is presented and it shows a good performance in front of other decentralized designs.

Resumen

Con el fin de proporcionar servicios inalámbricos para la demanda actual de las aplicaciones móviles de alta velocidad de datos, se necesitan con mayor eficiencia espectral. Es una realidad que los sistemas inalámbricos actuales están limitados por una división de la gestión del espectro de frecuencias que por un lado minimiza la interferencia multiusuario pero, por otro lado, impide el uso de señales con anchos de banda más amplios. Para tal fin, se plantena una reutilización de frecuencias más agresiva (a ser posible, todos los transmisores eventualmente podrían compartir la misma banda de frecuencia). Bajo este contexto, el uso de múltiples antenas para neutralizar la interferencia así como una asignación de potencia inteligente es primordial. Además, se requieren nuevas políticas de regulación del espectro para garantizar una convivencia pacífica entre las redes de espectro compartido.

El objetivo de esta tesis es proporcionar una conformación de haz y asignación de potencia de estos nuevos sistemas de espectro compartido que están destinados a aumentar exponencialmente la eficiencia espectral de los sistemas. Se proporciona un marco matemático basado en la optimización multicriterio para analizar la propuesta de conformación de haz, que sirve como una herramienta fundamental para la descripción de los estudios inciales en sistemas interferentes con múltiples antenas. En otras palabras, la región de tasas de comunicación para el sistema MISO interferente se describe y estudia. Además la conformación de haz óptima en suma de tasas de comunicación se estudia.

Por otro lado y con el fin de hacer frente al problema de altos niveles de interferencia en sistemas de espectro compartido, un nueva regulacón del espectro se revisa. En virtud de este mecanismo de gestión del espectro, un titular de la licencia es capaz de irradiar bajo una cierta porción de tiempo, dentro de un área concreta y en una banda dada. Por otra parte, la cantidad de potencia total (deasada e interferente) dentro de la zona, est'a limitada a un cierto valor. Dado que el control de los niveles de potencia en un área determinada es engorroso, proponemos restringir la potencia de recepción como una estimación de la potencia total acumulada. De este modo, se estudian los conformadores de transmisión óptima y la asignacín de potencias.

Resum

Per tal de proporcionar serveis sense fils per a la demanda actual de les aplicacions mòbils d'alta velocitat de dades, es necessiten sistemes amb major eficiència espectral. És una realitat que els sistemes inalambrics actuals estan limitats per una divisió de la gestió de l'espectre de freqüències que d'una banda minimitza la interferència multiusuari però, d'altra banda, impedeix l'ús de senyals amb amples de banda més amplis. Per a tal fi, es plantena una reutilització de freqüències més agressiva (idealment, tots els transmissors eventualment podrien compartir la mateixa banda de freqüència). Sota aquest context, l'ús de múltiples antenes per neutralitzar la interferència així com una assignació de potència intel·ligent és primordial. A més, es requereixen noves polítiques de regulacion de l'espectre per garantir una convivència pacífica entre les xarxes d'espectre compartit.

L'objectiu d'aquesta tesi és proporcionar una conformacion de feix i assignació de potència d'aquests nous sistemes d'espectre compartit que estan destinats a augmentar exponencialment l'eficiència espectral. Es proporciona un marc matemàtic basat en la optimització multicriteri per analitzar la proposta de conformacion de feix, que serveix com una eina fonamental per a la descripció dels estudis inicials en sistemes interferents amb múltiples antenes. En altres paraules, la regió de taxes de comunicació per al sistema MISO interferent es descriu. A més la conformació de feix òptima en suma de taxes de comunicació s'estudia i un diseny subòptim es presenta.

D'altra banda i per tal de fer front al problema d'alts nivells d'interferència en sistemes d'espectre compartit, un nova regulació de l'espectre es revisa. En virtut d'aquest mecanisme de gestió de l'espectre, un titular de la llicència és capaç d'irradiar sota una certa porció de temps, dins d'un àrea concreta i en una banda donada. Per dur això a terme, la quantitat de potència total (desitjada i interferent) dins la zona es limitada a un cert valor. Atès que el control dels nivells de potència en una àrea determinada és difícil, proposem restringir la potència de recepció com una estimació de la potència total acumulada . D'aquesta manera, s'estudien els conformadors de transmissió òptima i l'assignació de potències.

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Notation

Boldface upper-case letters denote matrices and boldface lower-case letters denote vectors.

$\mathbb{R}$, $\mathbb{C}$	The set of real and complex numbers, respectively.
$\mathbb{R}^{N\times M}$, $\mathbb{C}^{N\times M}$	The set of $N\times M$ matrices with real- and complex-valued entries, respectively.
$\lvert x\rvert$	Absolute value (modulus) of scalar x.
$\mathbf{a} \prec \mathbf{b}$	Componentwise less than.
$\lVert\mathbf{x}\rVert$	l^2-norm of vector $\mathbf{x}$, defined as $(x = \mathbf{x}^H\mathbf{x})^{\frac{1}{2}}$.
$[\mathbf{x}]_r$	The r-th vector element.
$[\mathbf{X}]_{r,c}$	The matrix element located in row r and column c.
$\mathrm{Tr}\{\mathbf{X}\}$	Trace of matrix $\mathbf{X}$. $\mathrm{Tr}\{\mathbf{X}\} = \sum_{n=1}^{N}[\mathbf{X}]_{n,n}$.
$\det(\mathbf{X})$	Determinant of matrix $\mathbf{X}$.
$\mathrm{diag}(\mathbf{x})$	A diagonal matrix whose diagonal entries are given by $\mathbf{x}$.
$\mathbf{I}$	Identity matrix. A subscript can be used to indicate the dimension.

$\mathbb{E}\{\cdot\}$ Statistical expectation.

w.r.t. With respect to.

Acronyms

AP	Access Point
AGC	Automatic Gain Control
ASA	Authorized Shared Access
AWGN	Additive White Gaussian Noise
DIR	Desired Impulse Response
DFE	Decision Feedback Equalizer
DOA	Direction Of Arrival
EC	European Commission
EIRP	Equivalent Isotropically Radiated Power
INR	Interference-to-Noise Ratio
LOS	Line-Of-Sight
LSA	Licensed Shared Access
LTE	Long Term Evolution
MDIR	Matched Desired Impulse Response
MF	Matched Filter
MIMO	Multiple-Input-Mingle-Output
MIN	Minimum Interference plus Noise
MISO	Multiple-Input-Single-Output
MOP	Multi-Objective Problem
MOLP	Multi-Objective Linear Program

MOLFP Multi-Objective Linear Fractional Problem

MURC Most Upper Right Corner

RLAN Radio Local Area Network

RF Radio Frequency

SIR Signal-to-Interference Ratio

SME Small-Medium Enterprises

QCQP Quadratically Constraint Quadratic Program

TRB Time Reference Beamforming

TAS Time-Area-Spectrum

UMTS Universal Mobile Telecommunications System

VS Virtual SINR

ZF Zero-Forcing

Chapter 1

Introduction

This thesis deals with the design of transmit beamforming and power allocation in spectrum sharing scenarios. In this context, multiple base stations try to send information to different receivers sharing time and frequency resources. These systems are restricted not only due to the multiuser interference but also due to the total amount of received power level which might incur in spectrum regulation violation. Since obtaining optimal designs results to be very computationally complex, we focus on suboptimal yet efficient solutions. Those solutions relay on realistic assumption which bring us the opportunity to solve difficult problems from an engineering perspective.

1.1 Motivation and Objectives

Due to the scarcity of the frequency and the current high data rate services user demands, more spectrally efficient wireless systems are investigated. A key aspect of these novel systems is a more aggressive frequency reuse so that a given frequency band supports several communication links pairs leading to a more efficient use of the spectrum. This strategy has been implemented in 802.11 communication services in LANs but; unfortunately, its extension to larger area networks is not straightforward.

When several base stations share the same frequency and time resources and its transmit power is not limited, the achievable data rates from all users are limited by the multiuser interference. Thus, the use of multiple antennas is desirable since their use is known to be very efficient for interference mitigation techniques. In contrast to the beamforming designs for the common broadcast channel, when several base stations transmit information in a spectrum sharing fashion, new optimization problems show up. Furthermore, as the received power level might become very high, a spectrum management regulation policy is compulsory in order to take the maximum benefit from each frequency band.

To sum up, the research lines of this thesis are the following:

- Study of the metrics and the mathematical model for designing an efficient beamformer in a spectrum sharing scenario.
- Design a beamforming for obtaining high achievable rates yet maintaining a low computational process.
- Design an efficient beamforming and power control design for spectrum sharing networks with regulatory constraints.

An individual chapter is dedicated to each of the above research lines and a brief summary of these chapters is presented in the next section.

1.2 Outline

This section provides an outline of this dissertation as well as a brief summary of each chapter.

Chapter 2

This chapter presents a general overview of spectrum sharing systems. Particularly, the historical wireless regulation is presented. First, the promotion of the shared use of the spectrum done by the FCC years ago is presented. The recent trends in cognitive radio are also depicted and; in addition, a technical criticism is explained. Indeed, the conservative regulation in terms of the opportunistic use of the spectrum, makes this communication model inefficient. On the other hand, recent trends on spectrum regulation are presented as well as the current status of those proposals.

Chapter 3

The MISO interference channel is presented and studied in this chapter. This communication model represents a spectrum sharing network where transmitters are equipped with multiple antennas and the receivers with a single one. The general capacity region of this channel is unknown but; however, the rate region when the receivers implemented single user detection has been parametrized by different authors in the current literature. We first show that those parametrizations correspond to the same multicriteria optimization problem.

Chapter 4

This chapter is devoted to characterize both physically and mathematically the use of transmit beamforming in spectrum sharing systems. First, the idea of beamforming directivity in interfered scenarios is reconsidered (i.e. taking into account that the radiated power levels to unintended receivers is self-defeating and it must remain as low as possible). Different notions of directivity are presented and evaluated but only one of them is elected considering its features. From another point of view, the beamforming design is presented as multicriteria optimization problem which perfectly models the underlying problem. In other words, the designer would like to have the maximum array gain to the intended receiver while nulling the radiated power to the nonintended ones. As we will see, this is an utopia point impossible to reach. In addition, the sum rate optimization is presented and solved under some assumptions. The resulting beamforming is presented in closed form and, as it is described in the chapter, corresponds to a reciprocal version of a reception design and the design that optimizes the presented notion of directivity.

Chapter 5

A novel regulation mechanism is studied in this chapter jointly with its optimal power control and beamforming design. In contrast to the current works, we propose to restrict the receive power level since the radiated power in transmitters with multiple antennas is impossible to obtain. Regulating the receive power of a wireless system serves as a mechanism for restricting the radiated power level autonomously and; in addition, a way for promoting the coexistence of different spectrum sharing networks geographically adjacent. The achievable rates are obtained and the optimal beamforming design is formulated. Unfortunately, this optimal design is computationally complex but we present an approximate solution that behaves well in all the transmit power range.

Chapter 6

This chapter concludes and summarizes the dissertation. Future research directions are envisaged as well as other interesting scenarios to be studied are presented.

1.3 Research Contributions

The contributions of this thesis has been presented in different international conferences as well scientific journals. They are listed here in their corresponding chapter.

Chapter 3

- M. Vázquez, A. Pérez-Neira, M. Lagunas, Multicriteria Beamforming in MISO Interference Networks, in Proceedings of IEEE Communication Theory Workshop (CTW 2011), 20-22 June, Sitges (Barcelona).

- M. Vázquez, A. Pérez-Neira, M. Lagunas, A Unifying Approach to Transmit Beamforming for the MISO Interference Channel, in Proceedings of International ITG Workshop on Smart Antennas (WSA 2012), 7-8 March 2012 Dresden, (Germany).

Chapter 4

- M. Vázquez, A. Pérez-Neira, M. Lagunas, Space-Time ML Receiver and its Reciprocal Transmitter Design for Interference Networks, in Proceedings of International ITG Workshop on Smart Antennas (WSA 2011), 24-25 February 2011 Aachen, (Germany).

- M. Vázquez, A. Pérez-Neira, M. Lagunas, Transmit Beamforming for the MISO Interference Channel with Asymmetric Link Gains, in Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2012) 25-30 March 2012, Kyoto (Japan).

- M. Vázquez, A. Pérez-Neira, M. Lagunas, Generalized Eigenvector for Decentralized Transmit Beamforming in the MISO Interference Channel , IEEE Transactions on Signal Processing , Vol. 61, No. 4 , pp. 878-882, February, 2013.

Chapter 5

- A. Pérez-Neira, M. Lagunas, M. Vázquez, Autonomous Design for the MISO Interference Channel with Mask Constraints at Reception, in Proceedings of the IEEE Applied Electromagnetics Conference AEMC, 18-22 December 2011, Calcutta (India).

- M. Lagunas, A. Pérez-Neira, M. Vázquez, Regulation and Research on Wireless Communications, in Proceedings of the IEEE Applied Electromagnetics Conference AEMC, 18-22 December 2011, Calcutta (India).

- M. Vázquez, A. Pérez-Neira, M. Lagunas, Open Spectrum Communication Systems and Optimized Downlink Beamforming, in Proceedings of 8th International Wireless Communications and Mobile Computing Conference (IWCMC 2012), 27-31 August, Cyprus.

- M. Vázquez, A. Pérez-Neira, M. Lagunas, Licensing Open Spectrum Systems, in Proceedings of 3rd International Workshop on Cognitive Information Processing (CIP), 28-30 May 2012, Parador de Baiona (Spain).

- M. Vázquez, A. Pérez-Neira, M. Lagunas, The Role of Received Power Constraints in Transmit Beamforming and Power Control Design for Spectrum Sharing Networks , IEEE Transactions on Signal Processing , submitted.

Other contributions not directly related with this dissertation

- M. Vázquez, T. Magesacher, Versatile Low PAPR and Low Out-Of-Band Power OFDM System , in Proceedings of 16th International OFDM-Workshop, 31-2 September 2011, Hamburg (Germany).
- M. Vázquez, A. Pérez-Neira, M. Lagunas, Confidential Communication in Downlink Beamforming, in Proceedings of the IEEE International Workshop on Signal Processing Advances for Wireless Communications (SPAWC 2012), 17-20 June 2012, Cesme (Turkey).
- P. Henarejos, M. Vázquez, G.Cocco, A. Pérez-Neira, Forward Link Interference Mitigation in Mobile Interactive Satellite Systems , in Proceedings of AIAA International Communications Satellite Systems Conference (ICSSC), 14-17 October 2013, Florence (Italy).
- V. Joroughi, M. Vázquez, B. Devillers, A. Pérez-Neira, Design of an Hybrid Ground/Space Architecture Based on Individual On-Board Antenna Power Optimization for The Forward Link of a Multi-Beam Broadband Satellite System , in Proceedings of AIAA International Communications Satellite Systems Conference (ICSSC), 14-17 October 2013, Florence (Italy).
- V. Joroughi, M. Vázquez, B. Devillers, A. Pérez-Neira, Design of an on-board Beam Generation Process for the Forward Link of a Multibeam Broadband Satellite System , in Proceedings of IEEE Global Communication Conference (Globecom), 9-16 December 2013, Atlanta (USA).

Chapter 2

Regulation in Spectrum Sharing Wireless Systems

A spectrum-sharing communication system is the one whose transmitters simultaneously transfer information sharing both time and frequency resources. The motivation of this communication structure is to extensively optimize the use of the spectrum; that it is to say, preclude the inefficient 'cake-cutting' fashion of the spectrum. Unfortunately, the shared use of the spectrum where several communication link pairs operate in the same frequency band, leads to a high multi-user interference power levels which severally decreases the achievable user data rates. Therefore, the system designer must focus on reduce these interfering signals so that the different communication links can coexist under a certain QoS restrictions.

So far, the primal motivation of the spectrum regulation was to preserve different interference-free frequency bands so that different services would coexist. From the beginning, the regulatory bodies individually sold the frequency bands to different users. This was rapidly changed to spectrum auctions which is the current system to assign the mobile broadband frequencies. Under these licenses, a wireless services provider has the right of radiating in a given frequency band during a long time period (typically around tens of years).

Despite this spectrum management[1] technique has opened the doors to the development of 3G and 4G mobile broadband services, a more efficient use of the spectrum is obtained with other techniques by means of the use of spectrum sharing policy. In the following, we describe these techniques and we present other ones which might also increase the spectrum efficiency in the near future.

[1]It is important to remark the different between radio resource management and spectrum management. The first term is related to the wireless system design which maximizes the data rates taking into account the generated interference. The latter refers to the spectral allocation done by the regulatory bodies in order to control the signal power strength in the different frequency bands.

2.1 Unlicensed

The unlicensed use of the spectrum was mainly motivated by the use of spread spectrum technology. These modulations were developed with the aim of reliable communications against jamming. It is important to remark that although the methods were studied years before separately (spreading sequences), it was in Vietnam war where they were used for the first time. Curiously, anti-jamming techniques are indeed an ideal mechanism for minimizing the generated interference to the non-intended users. Note that the problem becomes the same: provide a reliable communication in an interfered scenario, either this interference comes from a malicious agent or another user in the system.

By 1985, FCC was investigating novel regulation techniques in order enhance innovation in the telecommunication sector. This actually was a very important change in FCC politics: so far the innovation preceded the regulation. By that time, FCC expert Michel J. Marcus pointed out three possible technologies to be used, namely: millimetre waves, smart antennas and spread spectrum communications [34]. However, it was the last one that gained the attention of the other FCC members. After several deliberations, FCC promote the use of the 'unlicensed' spectrum as follows:

- **15.126 Operation of spread spectrum systems.** *Spread spectrum systems may be operated in the 902-928 MHz, 2400-2483.5 MHz and 5.725-5850 MHz frequency bands subject to the following conditions:*
  - *They may transmit within these bands with a maximum peak output power of 1 Watt.*
  - *RF output power outside these bands over any 100 kHz bandwidth must be 20 dB below that in any 100 kHz bandwidth within the band which contains the highest level of the desired power. The range of frequency measurements shall extend from the lowest frequency generated in the device (or 100 MHz whichever is lower) up to a frequency which is 5 times the center frequency of the band in which the device is operating.*
  - *They will be operated on a non-interference basis to any other operations which are authorized the use of these bands under other Parts of the Rules. They must not cause harmful interference to these operations and must accept any interference which these systems may cause to their own operations.*
  - *For frequency hopping systems, at least 75 hopping frequencies, separated by at least 25 kHz, shall be used, and the average time of occupancy on any frequency shall not be greater than four tenths of one second within*

a 30 second period. The maximum bandwidth of the hopping channel is 25 kHz. For direct sequence systems, the 6 dB bandwidth must be at least 500 kHz.

– *If the device is to be operated from the public utilities lines, the potential of the RF signal fed back into the power lines shall not exceed 250 microvolts at any frequency between 450 kHz and 30 MHz.*

Although the Wi-Fi technology appeared few years later, by that time some experts already suggest the possible implementation of a 'radio LAN' under this regulation. Curiously, the spread spectrum modulation technique solution did not triumph due to its bad behaviour to frequency selective channels. Indeed, there have been several changes of this regulation as for instance the allowance of using new waveforms such as OFDM and multiple antennas. The explosive growth of Wi-Fi devices was not expected by the time of the regulation was created. This FCC operation is considered one of the most important successes of the regulatory body since not only created a huge business but also because it extremely promote research in wireless communications. As an example, in 3.2 is shown the growth of WiFi transceivers integration in new electronic systems.

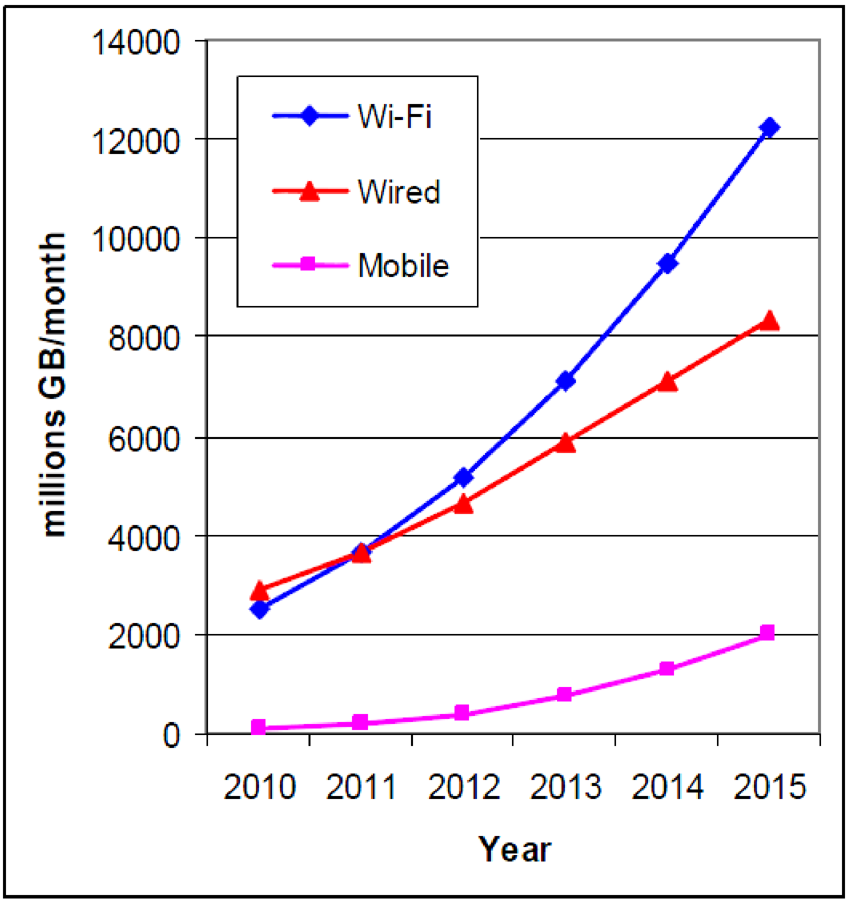

Figure 2.1: Growth of WiFi devices from Cisco internal report.

Currently also mobile wireless operators make use of this unlicensed use of the spectrum in order to leverage the accumulated traffic in the network. These methods are coined as data offloading and it is gained a lot of attention recently.

For the sake of the completeness, in the next table we also point out other frequency bands with unlicensed use.

Table 2.1: Frequency Band and Typical Use

40.660-40.700 MHz	Toys, Model control, Baby monitors
433,05-434,790 MHz	Radio Activated Key Entry (key fobs)
863-870 MHz	RFID, Cordless Audio, Industrial telemetry, Telecommand
5725-5875 MHz	CCTV, Wideband data
24.05 - 24.25 GHz	Movement detection
61.0-61.5 GHz	Not presently used - no generic equipment standard developed
122 - 123 GHz	Not presently used - no generic equipment standard developed
244 - 246 GHz	Not presently used - no generic equipment standard developed

2.2 Cognitive Radio

Cognitive radio techniques apply when a certain frequency band is already assigned to a determined usage but it is rather used. For example, this is the case of public safety of wireless services which are used rarely and some of the terrestrial TV broadcasting. In [35] an empirical study of the spectrum occupancy is presented and it is shown that in some cases there is a margin to be exploited. In order to cope with this disuse of certain frequency bands, spectrum managers has envisaged an opportunity for other unlicensed users by means of communicating in those frequency bands when they are not used.

This opportunistic use of the spectrum can be achieve by means of the use of cognitive radios [36]. The term cognitive radio refers to the radio transceiver that is able to obtain some information from its environment (frequency, modulation, power,...) and it adaptively changes its transmission accordingly. With this technique, an unlicensed (secondary) user might be able to establish whether or not a licensed (primary) user is using a given frequency band and; posteriorly, transmit in that band.

These frequency bands that are not used are coined as 'white spaces'. As a first attempt, FCC studied the use of those white spaces in the terrestrial TV band due to its propagation capabilities [33]. Later, they also licensed the use of the 5 GHz (i.e. the one used for radar applications).

In order to use those frequency bands, three different types of cognitive radio usage were indicated:

- **Spectrum sensing:** sense whether a frequency band is occupied or not by means of for example using cyclostationary properties of the signal.

- **Geolocation:** access a database where all the spectrum occupation is registered and, assuming that the transceiver is able to establish its position, check which frequency band is available for being used.

- **Beacons signals:** the availability of a given frequency band is published by an extra signalling.

Since the use of beacon signals imply a modification of the existing communication system, this last option was generally declined in the regulation process. On the other hand, the first two options are of great interest for cognitive radio application; and, with the aim of minimizing the eventual interference to the primary user, a combination of spectrum sensing and geolocation mechanism might be the best option for the experts.

In fact, the main issue to be addressed is how to maintain an interference free environment for the primary user yet providing a spectral efficient communication to the secondary ones. Note that interference can dramatically damage the wireless application (i.e. bad TV signal reception and of the false alarm in the radar system). Since this quality preservation is compulsory, FCC promoted a very restrictive use of the spectrum as for the 5 GHz which literally dictaminates:

- *The mechanism should be able to detect interference signals above a minimum detection threshold of -62 dBm for devices with a maximum EIRP of less than 200 mW and -64 dBm for devices with a maximum EIRP of 200 mW to 1 W averaged over 1 microsecond.*

- *A channel that has been flagged as containing a radar signal, either by a channel availability check or in-service monitoring, is subject to a 30 min period (non-occupancy period) where it cannot be used by the device in order to protect scanning radars. The non-occupancy period should start at the time when the radar signal is detected*

The aforementioned regulation permissions extremely restricts the cognitive use of the spectrum since note that this restriction is not only addressed to a single secondary user, but a set of them. Indeed, considering that more than one user can be employing the same frequency band leads to a very inefficient use of the spectrum assuming the aforementioned regulation.

2.3 Recent Trends

As the devices equipped with wireless connection are expected to still exponentially grow in the next years, there is a need of using the spectrum in a more efficient way in order to allocate those services. Therefore, novel spectrum management policies must be envisaged not only for increasing the spectral efficiency but also for fostering the business creation in the telecommunications area since it is known to be one of the most important economical agents of the future.

In 2012, EC spread a communication regarding future regulation for promoting spectrum sharing [10]. In that report, it was considered that the shared use of the spectrum will be the key stone for the next generation of wireless services and; concretely, there were identified some requirements for fostering the shared use of the spectrum:

- *Engaging mutual responsibility of users over acceptable limits of interference and appropriate mitigation strategies;*
- *Providing legal certainty on applicable rules and conditions, enforcement procedures as well as transparency about compatibility assumptions and protection rights;*
- *Incentivising investments in improved technologies beneficial for incumbents and additional users, while safeguarding and fostering competition;*
- *Identifying broad frequency channels for RLAN development as well as providing congestion forecasts to increase the predictability and reliability of the most important shared bands;*
- *Ensuring that any transition from exclusive rights of use to shared use enhances competition from additional users and in particular does not create undue competitive advantages for current or future right-holders.*

As it is shown, EC is truly considering its mission towards spectrum sharing systems not only as a promoting research and innovation but also for encouraging SMEs which are currently not able to provide wireless services due to the high price of the spectrum licenses, to enter into the spectrum sharing business. Two novel spectrum sharing policies are considered for sharing spectrum in the European region; namely, ASA and LSA.

ASA takes as starting point the cognitive use of the spectrum as we have described in the previous section and it devotes to improve the spectral efficiency by means of restricting the opportunistic use. Concretely, in contrast to the cognitive radio usage where secondary users are unknown and exempt to have a license, ASA determines that just a limited number of users can access a given frequency band in a opportunistic fashion. In addition, some collaboration between secondary users are expected via innovative cognitive radio method.

However, LSA is defined as

- *An individual licensed regime of a limited number of licenses in a frequency band, already allocated to one or more incumbent users, for which the additional users are allowed to use the spectrum (or part of the spectrum) in accordance with sharing rules included in the rights of use of the spectrum granted to the licenses, thereby allowing all the licenses to provide a certain level of QoS.*

In other words, LSA makes use of cognitive radio techniques to determine the availability of the frequency band and subsequently the secondary user establishes an agreement with the primary one which has to be reflect on the license agreement so that a determined QoS is guaranteed.

Another licensing systems to be in mind, it has been also studied the idea of extend the WiFi model to other services such as mobile and large are broadband communications. In order to do this, some modifications are needed. For instance, a lower frequency carrier might be advantageous due to its propagation loss as well as the use of higher transmit power.

Clearly, this new communication system will generate a very high interference power levels and the regulation which can avoid this situation it has not been conceived yet. Indeed, the classical radiated power restrictions seems not to apply in this scenario where an spectrum user might have more than one transmitter working in the same geographical area. Due to that, it must be envisaged some network level power restrictions so that different neighbouring licensed wireless providers can coexist. A proposal for this licensing system is shown in chapter V.

2.4 Conclusions

Regulation plays a key role in both industrial and research development. As a matter of fact, spectrum management policies have forested many wireless innovations as well as promoting new research activities. Due to this, governments have the duty of continuing rethinking how spectrum is allocated as well as research and innovation agents must both promote and guide these novel spectral policies. Clearly, future regulatory proposals will be extremely important in order to a more efficient use of the spectrum so that next generation high data rate wireless services will be available for the majority of citizens.

The next chapters are devoted to study the communication architectures created from the spectrum sharing regulation. Concretely, the ones that do not take into account an opportunistic use of the spectrum but a desynchronized one. Those communication paradigms can be embraced in the interference channel model whose sum-capacity and capacity region is still a challenge to the scientific community.

Chapter 3

Achievable Rates of the K-user MISO Interference Channel

Interference channel is the inherent model behind many practical problems where simultaneous transmissions take place, such as: open spectrum, multi-cell systems, etc. Those scenarios seek for high spectral efficiency since all the communication players share frequency and time resources. Therefore, interference becomes the bottleneck of the network performance and, therefore, it has to be diminished in order to provide a reliable communication for all the link pairs.

Transmit beamforming is potentially useful to reduce interference. However, obtaining the optimal precoders of the MISO-IC is known to be difficult. Until recently the achiveable rate region was not even parametrized [23, 37, 46, 60, 41]. Among those works, [41] is the one that obtains a design that can achieve the edge of the rate region (the so-called Pareto rate region) in a distributed fashion. Unfortunately, those designs are more theoretical than practical; due to both their computational complexity and their non-decentralized fashion, they would be difficult to implement in the future beyond-5G cellular systems.

As a matter of fact, centralized techniques require the exchange of information among nodes; thus, making its implementation complex. Conversely, decentralized[1] techniques result to be more practical and feasible (i.e. low channel state information exchange, non-synchronization requirements,...).

In this chapter we first revisit the achievable rates of the MISO-IC and we found a connection with the framework multicriteria optimization framework. Indeed, the transmit beamformers that achieve the Pareto rate region are obtained via solving the multiobjective array gain problem. Posteriously, we address the problem of the sum-rate maximization of the MISO interference channel when the transmitters do

[1] We coin the term *decentralized design* to describe a design where the transmitters do not have to exchange information. Notice that it is a more restrictive design than the distributed one, where in general the transmitters have to share a limited amount of information.

not share information. We derive this optimal beamformer design by considering practical power signal levels (i.e. we assume that the interference power level is below the desired and we also consider that the amount of interference is low both at the receiver side and the transmitter one). The resulting design is in closed-form which makes it easy to implement.

The rest of the chapter is organized as follows. Section II presents an a brief introduction of the multicriteria optimization theory. This vector optimization naturally explains different transmit array processing phenomena (e.g. the fact that it is impossible to jointly deliver the maximum array gain the intended user while nulling the radiated interference to the others). Section III presents the multicriteria optimization theory as a mathematical tool for studying the current results on characterization of the Pareto rate region of the MISO-IC. Furthermore, several details regarding the connection between the different methods are depicted.

3.1 Brief Overview of Multicriteria Optimization Problems

Since the multicriteria optimization theory plays a key role in the next sections as it serves as the mathematical tool for understanding the trade-offs not only in the rates of the spectrum sharing network but also in the beamforming design, we succinctly describe it here. For further details, the reader can address [14].

A multicriteria optimization problem can be formally expressed as

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \mathbf{f}(x) = (f_1(x), \ldots, f_K(x)) \\ \text{subject to} \quad & x \in \mathcal{X} \end{aligned} \tag{3.1}$$

where $f_k : \mathbb{C}^N \to \mathbb{R}$ for $k = 1, \ldots, K$ and in general $\mathcal{X}$ is given in a set of constraints. $\mathcal{Y}$ is defined as the set of all attainable points for all feasible solutions, $\mathcal{Y} = \mathbf{f}(\mathcal{X})$.

Definition 3.1. *A point $x \in \mathcal{X}$ is called Pareto optimal if there is no other x' such that $\mathbf{f}(x') \preceq \mathbf{f}(x)$.*

We will denote the set of all Pareto optimal points as $\mathcal{X}_E$ and their images as $\mathcal{Y}_E$. Sometimes, ensuring Pareto optimality for some problems is difficult. Due to this, the condition of optimality can be relaxed such as

Definition 3.2. *A point $x \in \mathcal{X}$ is called weakly Pareto optimal if there is no other x' such that $\mathbf{f}(x') \prec \mathbf{f}(x)$.*

There are several methods for finding the Pareto points of a MOP and they are coined as scalarization techniques. The election of a determined scalarization technique depends on the problem to be solved. For instance, the designer might chose the scalarization techniques that leads to a convex problem. On the other hand, it might interesting that the resulting scalarization method has physical meaning in order to provide more insights to the problem to be solved.

In the next subsections, three different scalarization techniques are described mathematically and their benefits are remarked. It is important to mention that each scalarization technique provides a set of Pareto points depending of a set of parameters that can be varied by the designer.

Weighted Sum Method

The simplest scalarization technique is the weighted sum method which collapses the vector objective sum into a single objective component sum:

$$\underset{x \in \mathcal{X}}{\text{minimize}} \quad \sum_{k=1}^{K} \lambda_k f_k(x) \tag{3.2}$$

where $0 \leq \lambda_k \leq 1$ and $\sum_{k=1}^{K} \lambda_k = 1$. With this, varying the set of λ_k values, any Pareto point might be obtain. However, this only happens in some cases. The next theorems relate the optimal solutions of (3.2) with the Pareto optimal points of the equivalent problem.

Theorem 3.1. *Let $\hat{x} \in \mathcal{X}$ be an optimal solution of* (3.2) *then $\hat{x}$ is weakly efficient.*

Proof. [14, Proposition 3.9.] □

Theorem 3.2. *Let $\mathcal{X}$ be a convex set and $f_k, k = 1, \ldots, K$ be convex functions, if $\hat{x}$ is weakly efficient then there is some set of $\lambda_k \quad k = 1, \ldots, K$ such that $\hat{x}$ is an optimal solution of* (3.2)*.*

Proof. [14, Proposition 3.10.] □

Therefore, convexity is required for achieving weakly Pareto optimal points with the weighted sum method. This fact makes the weighted-sum method inconvenient for several MOP. Fortunately, it is possible to relax the convexity condition in order to attain more optimization problems. We first describe the notion of directional convex condition.

Definition 3.3. *Given a non-zero vector $\mathbf{p} \in \mathbb{R}^{K \times 1}$, $\mathcal{Z} \subset \mathbb{R}^{K \times 1}$ is said to be $\mathbf{p}$-directionally convex if given two different points in $\mathcal{Z}$, $\mathbf{z}_1, \mathbf{z}_2$, and two positive scalars, α_1, α_2, with $\alpha_1 + \alpha_2 = 1$, there is a positive number β such that $\alpha_1 \mathbf{z}_1 + \alpha_2 \mathbf{z}_2 + \beta \mathbf{p} \in \mathcal{Z}$.*

Notice that a convex set is actually $\mathbf{p}$-directionally convex for any $\mathbf{p}$. Under the assumption of a set to be $\mathbf{p}$-directionally convex, we can formulate the necessary and sufficient conditions of the weighted-sum method to achieve

Theorem 3.3. *The solutions of the weighted-sum method are Pareto optimal if and only if the set $\mathcal{Y}$ is* $\mathbf{p}$*-directionally convex.*

Proof. [30, Theorem 1(b)] $\square$

With this we have characterized the weighted-sum scalarization method that can easily be used in case the aforementioned holds. However, the λ_k values might be difficult to set a priori since it is difficult to provide a physical meaning related to the problem to be optimized. The next scalarization technique solves this problem by incorporating constraints to the objective vector function.

Epsilon-constraint Method

With this method, there is no aggregation of criteria, instead only one of the original objectives is minimized while the others are transformed into constraints.

$$\begin{aligned} &\underset{x\in\mathcal{X}}{\text{minimize}} \quad f_j(x) \\ &\text{subject to} \quad f_k(x) \leq \epsilon_k \quad k=1,\dots,K \quad k\neq j \end{aligned} \tag{3.3}$$

Contrary to the weighted-sum method, with this scalarization technique no convexity assumptions are needed for reaching all Pareto optimal points as the next theorem describes.

Theorem 3.4. *Let $\hat{x}$ be an optimal solution of* (3.3) *for some j. Then $\hat{x}$ is weakly Pareto optimal.*

Proof. [14, Proposition 4.3.] $\square$

The necessary conditions are obtained in the same way.

Theorem 3.5. *The feasible solution $\hat{x} \in \mathcal{X}$ is Pareto optimal if and only if there exists a set of ϵ_k, $\quad k = 1\dots,K$ such that $\hat{x}$ is an optimal solution of* (3.3) *for all $j = 1,\dots,K$.*

Proof. [14, Proposition 4.5.] $\square$

With this, with appropriate choices of ϵ_k all optimal points are achievable even if $\mathcal{Y}$ is not convex. In addition, ϵ_k values have a physical meaning as they are have the same units as the component of the vector objective function. For instance, if the vector objective function represents the benefits of different goods, obtaining the Pareto region of the benefit set can be obtained by maximizing one of the good while restricting the others to a given value.

Weighted Chebychev Sum Method

This procedure transforms the multicriteria optimization problem into

$$\underset{x\in\mathcal{X}}{\text{minimize}} \quad \underset{k=1,\ldots,K}{\text{maximize}} \quad \omega_k \left(f_k(x) - y_k^U\right) \tag{3.4}$$

where y_k^U is the minimum value of the objective function $f_k(x)$, coined as utopia point. Note that the method is based on a distance minimization (i.e. the distance between each objective vector component and its utopia point). Indeed, the weighted chebychev sum method penalizes the vector component with its highest distance with the utopia point.

As the next theorem describes, this method can attach all the Pareto points.

Theorem 6 [15, Theorem 4.24.]: A feasible solution $\hat{x} \in \mathcal{X}$ is weakly efficient if and only if there exists a set of $\omega_k, \quad k = 1\ldots, K$ such that $\hat{x}$ is an optimal solution of (3.4).

So far, a multicriteria optimization mathematical framework has been presented. Now it is time to use this technique for describing the achievable rates of the wireless spectrum sharing network. As we will describe in the next sections, the problem of obtaining the optimal achievable rates of the MISO interference channel can be cast as a MOP.

3.2 Pareto Rate Region of the MISO-IC

3.2.1 System Model and Problem Statement

As the focus of this chapter is to obtain the achievable rates of the MISO-IC, we now proceed to formalize this problem.

Let us consider a multiuser wireless system with K communication link pairs, where each transmitter is equipped with N antennas and wants to transmit information to its desired received. The signal at receiver k is given by

$$y_k = \sum_{j=1}^{K} \mathbf{h}_{jk}^H \mathbf{b}_j \sqrt{P} s_j + n_k \tag{3.5}$$

where $\mathbf{h}_{jk} \in \mathbb{C}^{N\times 1}$ is the vector channel from transmitter j to receiver k, $\mathbf{b}_j \in \mathbb{C}^{N\times 1}$ is the unit norm transmit beamformer of user k. P is the transmitted power assumed to be the same and fixed for all users, s_k is the zero mean unit variance data symbol sent from transmitter k to its intended receiver and n_k is the zero mean Gaussian noise with variance σ^2.

The achievable rate of the k-th user when it treats interference as noise is

$$R_k = \log\left(1 + \frac{P|\mathbf{h}_{kk}^H \mathbf{b}_k|^2}{\sum_{j\neq k} P|\mathbf{h}_{jk}^H \mathbf{b}_j|^2 + \sigma^2}\right). \quad (3.6)$$

Under this context, obtaining the Pareto rate region of the MISO-IC (i.e. its optimal rate tuples) can be formulated as a multicriteria optimization problem

$$\begin{aligned} \underset{\{\mathbf{b}\}_{k=1}^K}{\text{maximize}} \quad & \mathbf{R} \\ \text{subject to} \quad & \|\mathbf{b}_k\| = 1 \quad k = 1, \ldots, K \end{aligned} \quad (3.7)$$

where

$$(\mathbf{R})_k = R_k. \quad (3.8)$$

Clearly, solving the MOP (3.7) leads to the edge of the rate region of the MISO-IC when receivers treat interference as noise. Consquently, the plethora of existing works regarding the obtaintion of the MISO-IC optimal rate tuples must be no more than a scalarization technique of (3.7).

3.2.2 Rate Profiling

It is easy to show that the technique presented [41] for obtaining the Pareto rate region is in fact an scalarization technique of (3.7). This method is described as

$$\begin{aligned} \underset{\{\mathbf{b}_k\}_{k=1}^K}{\text{maximize}} \quad & R \\ \text{subject to} \quad & \|\mathbf{b}_k\|^2 = 1 \quad k = 1, \ldots, K \\ & r_i \geq \nu_k R \quad k = 1, \ldots, K \end{aligned} \quad (3.9)$$

where

$$\nu_k \geq 0 \quad \sum_{k=1}^K \nu_k = 1 \quad (3.10)$$

In [41] is shown that this problem is nonconvex but it can be solved by series of SOCP feasibility problems. This problem can be reformulated as

$$\begin{aligned} \underset{\{\mathbf{b}_k\}_{k=1}^K}{\text{maximize}} \quad & R \\ \text{subject to} \quad & \|\mathbf{b}_k\|^2 = 1 \quad k = 1, \ldots, K \\ & \frac{r_k}{\nu_k} \geq R \quad k = 1, \ldots, K \end{aligned} \quad (3.11)$$

where it has been assumed that $\nu_k > 0 \quad k = 1, \ldots, K$ which does not incur in any lost of generality since when $\nu_k = 0$ the constraint does not longer exist. The optimization problem (3.11) is the same as

$$\begin{aligned} \underset{\{\mathbf{b}_k\}_{k=1}^K}{\text{maximize}} \quad & \underset{k=1,\ldots,K}{\text{minimize}} \quad \frac{r_k}{\nu_k} \\ \text{subject to} \quad & \|\mathbf{b}_k\|^2 = 1 \quad k = 1, \ldots, K \end{aligned} \tag{3.12}$$

This last optimization problem is a weighted Chebychev scalarization of (3.7) MOP. Note that, (3.9) is computationally complex and in order to obtain a given Pareto point, the transmitters need to exchange information. This fact, motivates the search for a distributed algorithm so that each transmitter can eventually determine its beamforming vector, leading to an optimal point of the rate region. The next derivation aims to this purpose.

3.2.3 Array Gain Pareto Region

The transmit beamforming design has an important trade-off when considering the array gain. Indeed, it is not possible to jointly obtain the maximum array gain while nulling the radiated interference to the non-intended receivers. Consquently, the transmit beamforming optimization is again a MOP. Let us consider the beamforming of a transmitter in a MISO IC as a MOP. The problem can be formulated as follows

$$\begin{aligned} \underset{\mathbf{b}_k}{\text{maximize}} \quad & \mathbf{\Gamma}_k \\ \text{subject to} \quad & \|\mathbf{b}_k\|^2 = 1 \end{aligned} \tag{3.13}$$

where $\mathbf{\Gamma}_k$ is the vector objective function (3.14)

$$\mathbf{\Gamma}_k = \left(-\|\mathbf{b}_k^H \mathbf{h}_{k1}\|^2, -\|\mathbf{b}_k^H \mathbf{h}_{k2}\|^2, \ldots, \|\mathbf{b}_k^H \mathbf{h}_{kk}\|^2, \ldots, -\|\mathbf{b}_k^H \mathbf{h}_{kK}\|^2\right). \tag{3.14}$$

The utopia solution would be a transmit beamformer that nulls the power radiated to the interfered receivers (zero forcer) while maximizing the power transmitted to the intended one (matched filter). This utopia point is impossible to obtain and, therefore, the designer has to make a decision according to his/her preferences.

Recalling the previous section for user k we have that $\mathcal{X}$ are the beamformers whose norm is equal to P_k, $\mathcal{Y}$ is the set containing all the possible transmit gains, $\mathcal{Y}_E$ is the Pareto gain region and $\mathcal{X}_E$ are the optimal beamformers that attain all possible Pareto points of the power gain region $\mathcal{Y}_E$.

The importance of this new MOP was pointed out in [37, Theorem 2] which is the reference result to easily derived the fact that a rate Pareto optimal point can be achieved if all transmitters design their beamformers as optimal solutions of (3.13).

With this, we can focus our attention to solve (3.13). First, we can use the

ϵ-constraint method

$$\begin{aligned} \underset{\mathbf{b}_k}{\text{maximize}} \quad & \Gamma \\ \text{subject to} \quad & \|\mathbf{b}_k\|^2 = 1 \\ & \Gamma \leq \epsilon_k \quad j \neq k \end{aligned} \tag{3.15}$$

On the other hand, we can make use of the weighted-sum method

$$\begin{aligned} \underset{\mathbf{b}_k}{\text{maximize}} \quad & \sum_{j=1}^{K} \lambda_j^k \Gamma_j \\ \text{subject to} \quad & \|\mathbf{b}_k\|^2 = 1 \end{aligned} \tag{3.16}$$

In this case, all optimal values can be obtained since it is easy to show that the image of $\mathbf{\Gamma}$ is $\mathbf{p}$-directionally convex. Using properties of Hermitian matrices, the optimal values of (3.16) are obtained as

$$\mathbf{b}_k = \sqrt{P}\mathbf{v}_{min}\left(\sum_{j\neq k}^{K} \lambda_j^k \mathbf{h}_{kj}\mathbf{h}_{kj}^H + (1 - \sum_{j=1}^{Q} \lambda_j)\mathbf{h}_{kk}\mathbf{h}_{kk}^H\right) \tag{3.17}$$

Both with (3.15) and (3.17) we can obtain *all* possible array gains. In other words, any transmit[2] beamforming design must obtain either the appropriate λ or ϵ, depending on the optimization problem. Note that there are some intrinsic differences between both techniques: where as in (3.15) it is clear the physical meaning of ϵ, that it is not the case for the (3.17) method. Unfortunately, (3.15) is not expressed in closed from and it might be difficult and complex to compute in some cases (when the semidefinite programming relaxation fails). That it is not the case of the weighted-sum method where as it is expressed in (3.17) only needs an eigen-decomposition.

With this last results we can show that [37, 46] are different scalarization techniques of the same optimization problem (3.13) that can achieve the Pareto optimal rate region as it was proved in those works. Given an optimal array gain solution, it can be connected via considering the results in the appendices which connect the scalarization techniques presented in the previous section. For the sake of completeness, we describe the different methods.

Finally, (3.17) can be rewritten as

$$\mathbf{b}_k = \eta\left(\sum_{j\neq k} \lambda_j^k \mathbf{h}_{kj}\mathbf{h}_{kj}^H + \lambda_k^k \mathbf{I}\right)^{-1} \mathbf{h}_{kk} \tag{3.18}$$

which is the parametrization presented in [60].

[2]Eventually a receiver beamforming can be also designed under this framework

3.2.4 Simulation Results

This sections aims to compare weighted sum and epsilon constraint techniques. We consider a two user scenario each transmitter equipped with $N = 2$ antennas. We consider a unit norm single channel realization. In the case of the epsilon constraint method, we use the semidefinite relaxation technique, which for this case of a single quadratic constraint, provides the same optimal solution of the non-relaxed problem depending on the number of constraints [32]. In the case of the epsilon constraint method, we use the semidefinite relaxation technique, which for this case of a single quadratic constraint, provides the same optimal solution of the non-relaxed problem. For the simulation we use `CVX`, a package for specifying and solving convex programs [20].

Fig. 3.1 shows the array gain region for the user 1. It is clear that both the weighted and the epsilon-constraint scalarization techniques achieve all possible rates. Moreover, when the rate is evaluated (Fig. 3.2), both techniques can achieve the Pareto rate region.

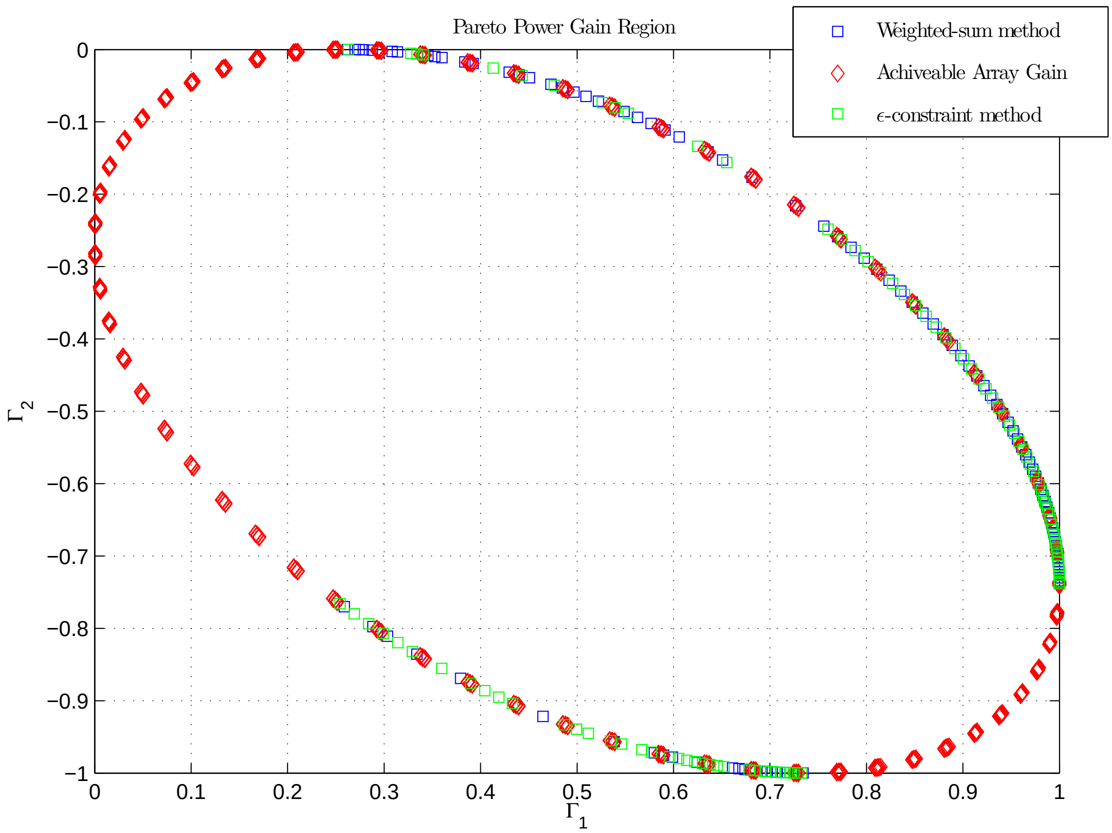

Figure 3.1: Power Gain Region: $\mathbf{\Gamma}_1$

It is important to remark that the red points (i.e. all possible beamforming vectors and their corresponding rate values), have been obtained in a grid search of

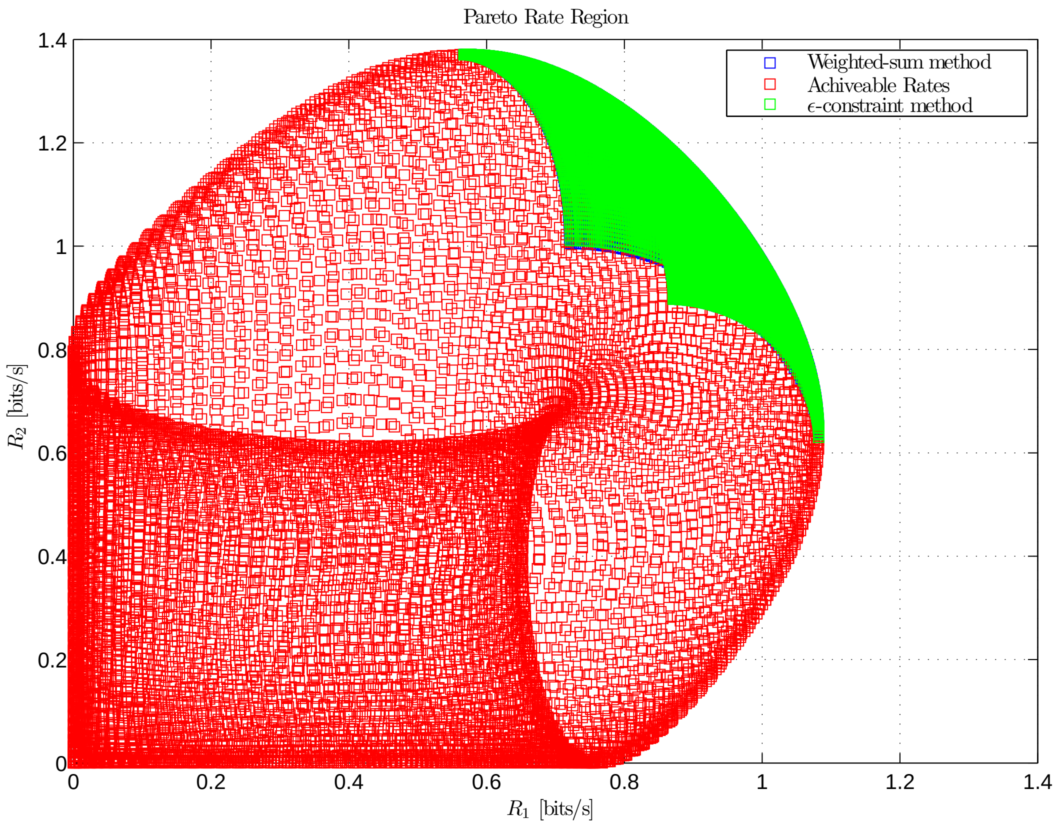

Figure 3.2: Rate region

complex 2-dimensional vectors with unitary norm.

3.3 Summary

The MISO-IC is an important mathematical model that represents the common spectrum sharing wireless network. The network optimization results to be very difficult not only due to the computational complexity but also due its decentralized nature (transmitter do not share information). Despite this, we found a mathematical framework that encompasses all the current characterizations of the rate region. Unfortunately, these characterizations are difficult to implement in real systems and; thus, in the next chapter we focus our attention in the beamforming design that might eventually not reach the Pareto rate region, but it provides efficient sum-rate solutions while preserving a low computational complexity.

It is important to remark the contribution of the presented chapter. We have shown that the efficient design of a transmit beamformer in spectrum sharing communications is reduced to the computation of $K-1$ parameters. The computation of those parameters can be obtained via a centralized design or, on the other hand, to be distributely computed by each of the transmitters. These parameters can take different forms and values but, as we described in the appendices, they represent the same notion of controlling the array gain of the transmit beamformer.

Appendix 3.A Relation between ϵ and ω

Bearing in mind the two scalarization techniques, namely, the ϵ-constraint method

$$\begin{aligned} \underset{x\in\mathcal{X}}{\text{minimize}} \quad & f_k(x) \\ \text{subject to} \quad & f_i(x) \leq \epsilon_i \quad \forall i \neq k \\ & x \in \mathcal{X} \end{aligned} \tag{3.19}$$

and the weighted Chebyshev norm

$$\begin{aligned} \underset{x\in\mathcal{X}}{\text{minimize}} \quad & \underset{k=1,\ldots,K}{\text{maximize}} \quad \omega_k \left(f_k(x) - y_k^U \right) \\ \text{subject to} \quad & x \in \mathcal{X} \end{aligned} \tag{3.20}$$

we want to show the equivalence between these two methods. According to [31] section III, (3.20) is equivalent to

$$\begin{aligned} \underset{x\in\mathcal{X}}{\text{minimize}} \quad & \alpha \\ \text{subject to} \quad & \alpha \geq \omega_k \left(f_k(x) - y_k^U \right) \\ & x \in \mathcal{X} \end{aligned} \tag{3.21}$$

On the other hand, we can introduce an additional variable in (3.22) and then the problem becomes

$$\begin{aligned} \underset{x\in\mathcal{X}}{\text{minimize}} \quad & t \\ \text{subject to} \quad & x \in \mathcal{X} \\ & f_i(x) \leq \epsilon_i \quad \forall i \neq k \\ & f_k(x) \leq t \end{aligned} \tag{3.22}$$

It is clear that the equivalence between the two methods is

$$\epsilon_i = \frac{\alpha}{\omega_i} + y_k^U \tag{3.23}$$

Appendix 3.B Relation between ϵ and λ

Given a Pareto point ($\mathbf{x}^*$), it can be obtained either by the weighted sum method or the epsilon constraint one (assuming that all the Pareto set is achievable by both methods). Therefore, there is a relation between $\{\lambda_k\}_{k=1}^K$ and $\{\epsilon\}_{k=1}^K$:

- Given a solution obtained by the weighted sum,$\mathbf{x}^*_{\mathrm{ws}}$, the same solution can be obtained by means of setting

$$\epsilon_k = f_k(\mathbf{x}^*_{\mathrm{ws}}) \quad k = 1, \ldots, K \tag{3.24}$$

- Alternatively, a solution obtained by the epsilon constraint method,$\mathbf{x}^*_{\mathrm{ec}}$, is obtained setting $\{\lambda_k\}_{k=1}^{K}$ as the Lagrange multipliers for the constraint objectives.

Chapter 4

Efficient Transmit Beamforming in the Interference Channel

The use of array processing techniques gained a lot of attention years ago in the context of wireless signal reception both from academy and industry. For instance, [27] presented a joint array and sequence detector for improving the data rates in UMTS systems. Under that receiver scheme, the beamforming is devoted to mitigate the unwanted signals (i.e. both co-channel interferences and late arrivals). Another example of beamforming design for wireless signal reception can be found in [44] were again the receiving beamforming plays the role of interference and multipath signal mitigation but for this case in the context of positioning systems.

Although adding more RF chains to a base station can be financially profitable due to its tentative increase on data rate, this might not be the case for general wireless devices (smartphones, M2M modules, etc.) where the use of additional RF chains increases the terminal cost. Therefore, this precludes (or makes it very difficult) the use of array processing methods in handle wireless receivers. Indeed, the use of multiple antennas in user terminals was not consider before the latests versions of LTE (LTE Advanced) not only due to the implementation difficulties but also for the cost increase.

An interesting option is to transfer the multi-antenna processing from the receiver to the transmitter in order to maintain the final user device with a low cost. This copernican revolution where the multiple antennas are shifted from the receiver to the transmitter can ideally substitute the well-known receive interference mitigation techniques yet maintaining a low complex single antenna[1] receiver. It is important to remark that spectrum sharing communication system where multi-user interference becomes the communication bottleneck, array processing techniques are specially

[1]In this thesis we consider that for each antenna there is a correspoding RF chain. Note that recently there are some works that although multiple antennas are consider, the communication techniques are conceived considering that there is only a single RF chain.

needed since unwanted signals (interferences for this case) are dominant.

Transmit array processing techniques entail a more challenging problem than the receiver ones. First, obtaining the receiver spatial signature is not longer a task that consists only of a channel estimation but also a feedback communication. This information transfer is always difficult and it implies an implicit delay and errors due to the limited rate feedback channel. Secondly, whereas the receiving spatial filtering design is generally derived via estimation theory, transmit beamforming involves the use of other theories that sometimes yield to counterintuitive engineering results.

This chapter aims to provide two frameworks for the study of transmit array processing techniques. In the first section, we present a novel evaluation of transmit beamforming techniques from antenna engineering perspective which involves to revisit the idea of directivity in a multiuser scenario where note that the radiated interference should be also considered as wasted energy. Furthermore, we formulate the optimization of the sum rate and we indentify that entails complex computations and a central unit processing. In order to solve this problem, we assume that the interference has a lower power level with respect to the desired signal. This assumption jointly with other ones, bring us the opportunity to come up with a decentralized transmit beamformer. Finally, we connect this novel beamformer with another receive one that incorporates AGC restrictions.

4.1 Review of Antenna Array Efficiency in Multiuser Communications

4.1.1 Classical Antenna Array Directivity

The directivity of an antenna is defined as *the ratio of the maximum radiation intensity to the radiation intensity averaged over all directions.* Mathematically,

$$D_{\text{antenna}} = \frac{4\pi U_{\max}}{P_{\text{rad}}} \tag{4.1}$$

where U is the radiation intensity (W/unit solid angle) and P_{rad} is the total radiated power. When considering not only a single antenna but a set of them, the beamforming effect must be taken into account so that the directivity for that case becomes

$$D_{\text{array}} = \frac{4\pi|\mathbf{w}^H\mathbf{s}_{\max}|^2}{|\mathbf{w}^H\mathbf{w}|^2} \tag{4.2}$$

where $\mathbf{w}$ denotes the transmit beamforming vector and it has been assumed that the array elements are isotropic.

Given a single user scenario where the transmitter is equipped with multiple antennas and it has total radiated power restriction $P_{\max}$, it is clear that for maximizing the array directivity, the designer has to chose the transmit beamforming so that

$$\mathbf{w}^* = \sqrt{P_{\max}}\mathbf{s}_{\text{user}} \tag{4.3}$$

where $\mathbf{s}_{\text{user}}$ denotes the steering vector of the user. The name of this transmit beamforming scheme is matched filter.

This toy example provides very useful insights. First, it appears that (4.3) also optimizes the receiver SNR since it is

$$\text{SNR} = \frac{P|\mathbf{w}^H\mathbf{s}_{\text{user}}|^2}{\sigma^2} \tag{4.4}$$

where P is the transmit power and we have assumed that the noise level at the receiver is σ^2 and there is no channel gain. It is remarkable how these two engineering paremeters (directivity and SNR) are related. Moreover, as it happens for the AWGN channel the maximization of the SNR yields to the channel capacity. Therefore, for this scenario, optimizing the array directivity implies obtaining the channel capacity of the MISO Gaussian channel with total power constraint [50].

Unfortunately, when more than one user is considered this relationship between the antenna array and the achievable rates does not longer exist. In fact, the capacity-achievable scheme entails complicated operations of interference presubstraction coined as dirty paper coding method [11, 53]. Without considering this complex scheme yet optimal, next section focuses on how to again redefine the physical meaning of antenna directivity in multi-user scenarios in order to again link the both interpretations.

4.1.2 Multi-user Antenna Array Efficiency

Although our focus on this work is to redefine the notion of directivity, we will not use this term so that any misunderstanding is minimized. To this end, we will use the term *multiuser antenna array efficiency* so that we consider a transmitter equipped with an antenna array that sends information to an intended user in a given time instant and interferences other users that are operating in the same frequency band. The non-intended users have spatial signature equal to $\mathbf{s}_i^q \quad q = 1, \ldots, Q$ where it has been assumed that there are Q interfered users.

Bearing this in mind, let us consider the following figure of merit

$$\eta_1 = \frac{|\mathbf{w}^H\mathbf{s}_{\text{user}}|^2}{\sum_{q=1}^{Q}|\mathbf{w}^H\mathbf{s}_i^q|^2 + \alpha|\mathbf{w}^H\mathbf{w}|^2} \tag{4.5}$$

where the denominator has been penalized interpreting that the radiated power delivered to the non-intended user $\left(\sum_{q=1}^{Q} |\mathbf{w}^H \mathbf{s}_i^q|^2\right)$. Furthermore, the total radiated power $|\mathbf{w}^H \mathbf{w}|^2$ is multiplied by a factor α which takes into account the antenna losses. With this new metric, it is clear that the wasted radiation energy is taken into account and it is a more realistic measurement of the antenna array efficiency in multi-user scenarios.

Nevertheless, η_1 may not properly reflect the array antenna efficiency since the denominator takes into account the same radiated power two times (i.e. the interfering transmit power is also included in the total radiated power). Due to that, it can be beneficiary recalling the idea of retro-directive antennas from radar systems whose efficiency can be described as

$$\eta_2 = \frac{|\mathbf{w}^H \mathbf{s}_{\text{user}}|^2 - \sum_{q=1}^{Q} |\mathbf{w}^H \mathbf{s}_i^q|^2}{\alpha |\mathbf{w}^H \mathbf{w}|^2} \tag{4.6}$$

where in this case, the radiated interfering power is substracted to the desired one. Note that for η_2 there might be the case this efficiency becomes negative and, therefore, it does not represent the common efficiency value. In order to solve this problem, we propose a metric which is a combination of the aforementioned multi-user efficiencies (η_1, η_2)

$$\eta = \frac{|\mathbf{w}^H \mathbf{s}_{\text{user}}|^2 - \sum_{q=1}^{Q} |\mathbf{w}^H \mathbf{s}_i^q|^2}{\sum_{q=1}^{Q} |\mathbf{w}^H \mathbf{s}_i^q|^2 + \alpha |\mathbf{w}^H \mathbf{w}|^2} = \frac{|\mathbf{w}^H \mathbf{s}_{\text{user}}|^2 + \alpha |\mathbf{w}^H \mathbf{w}|^2}{\sum_{q=1}^{Q} |\mathbf{w}^H \mathbf{s}_i^q|^2 + \alpha |\mathbf{w}^H \mathbf{w}|^2} - 1 \tag{4.7}$$

For these different multiuser efficiency metrics, we now derive its corresponding optimal beamforming design.

Theorem 4.1. *The transmit beamforming that optimizes η_1 and η_2 is*

$$\mathbf{w}_{\textit{Virtual SINR}} = \left(\sum_{q=1}^{Q} \mathbf{s}_q^i (\mathbf{s}_q^i)^H + \alpha \mathbf{I} \right)^{-1} \mathbf{s}_{user} \tag{4.8}$$

Proof. The derivation can be found in Appendix 4.A. □

we name this beamformer as *Virtual SINR* as it will be explained in the next chapter since the already presented beamforming structure has been presented several times in the literature. When optimizing *eta* instead, the resulting beamformer is

Theorem 4.2. *The transmit beamforming that optimizes η*

$$\left(\mathbf{s}_{user} \mathbf{s}_{user}^H + \alpha \mathbf{I}\right) \mathbf{w}_{EIG} = \lambda_{max} \left(\sum_{q=1}^{Q} \mathbf{s}_q^i (\mathbf{s}_q^i)^H + \alpha \mathbf{I} \right) \mathbf{w}_{EIG} \tag{4.9}$$

Proof. The derivation can be found in Appendix 4.A. □

In this case, (4.9) corresponds to another transmit beamforming structure that cannot be found in the literature. We coined EIG beamforming at it will also appear the next chapters in different forms. It is important to metion that these transmit beamforming designs are the reciprocal to the ones presented in the previous section.

In the next subsection we evaluate the use of this metric for evaluating transmit beamforming designs. Both (4.8) and (4.9) will be evaluated jointly with the classical transmit beamforming designs (i.e. zero forcing and matched filter).

4.1.3 Simulation Results

In order to evaluate both definitions, it is considered a transmitter equipped with 4 antennas. The receiver is positioned in the broadside direction (0 degrees) while the interfered receiver is first situated at 0 degrees an it is moved till a position orthogonal to the broadside direction (90 degrees). For a fair comparison, we consider that η_1 is normalized by a factor of $1/P$ and η_2 by a factor of $\frac{1}{P+1}$.

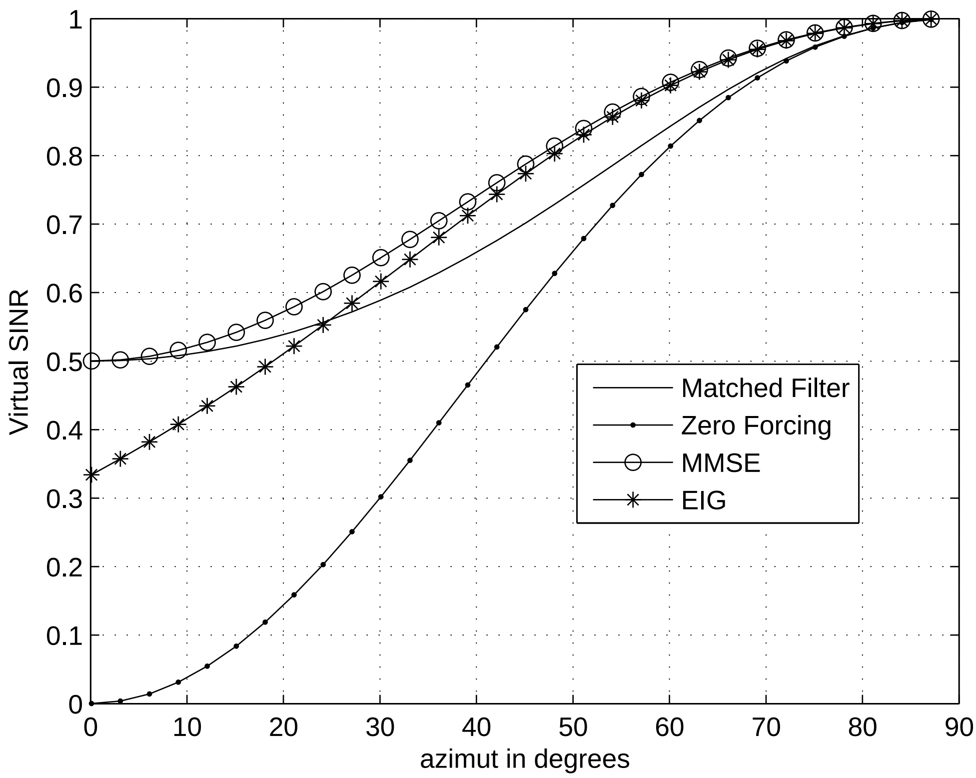

Figure 4.1: η_1 versus interference angle of arrival, $P = 0dB$

Figures 4.1 and 4.2 show the behaviour of η_1 measure. Clearly, when the transmitted power is low MMSE beamformer outperforms EIG. On the other hand, when the transmitted power is high, EIG is better.

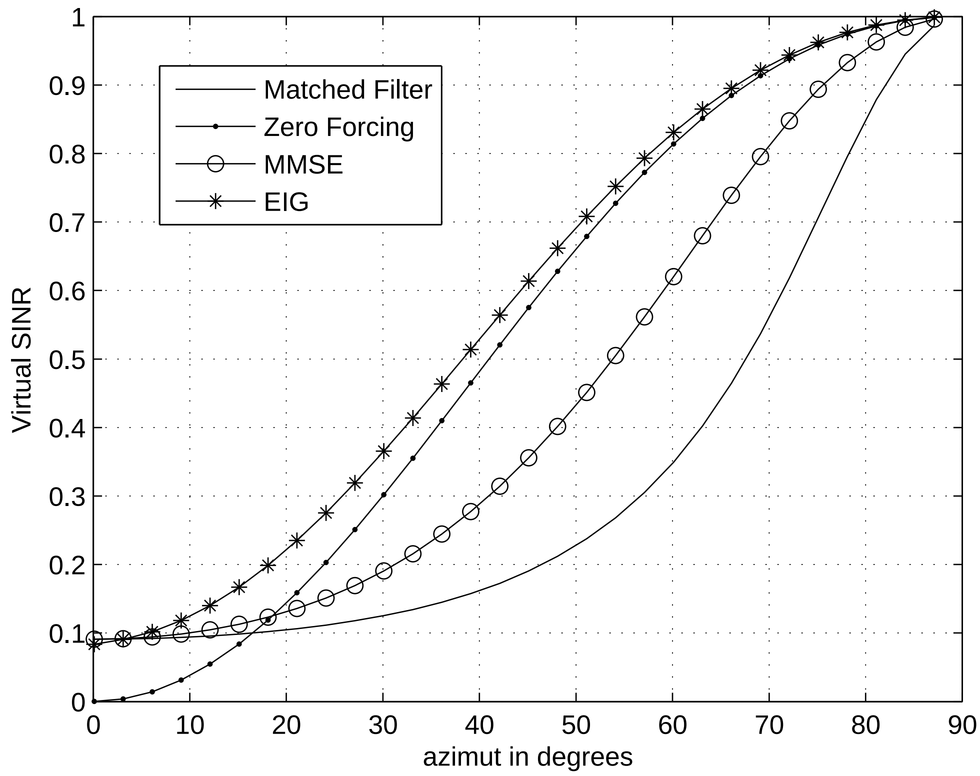

Figure 4.2: η_1 versus interference angle of arrival, $P = 10dB$

Figures 4.3 and 4.4 show the η ratio. At a first glance, we can see that the curves remain the same altough the transmitted power is varied. The worst behaviour is shown by the matched filter followed by the zero forcing. Finally EIG has even more better directivity than MMSE. Notice how the zero forcing technique outperforms the MMSE when the transmitted power is high. In both cases, EIG show the best behaviour because it actually maximizes η. In any case, it is clear that η is the best figure of merit since its range value goes from 0 to 1 and there are no crosses between the different curves.

The notion of directivity serves as a guide to obtain efficient transmit beamformers. As we will see in the next section, the already presented designs are also sum-rate optimal in some scenarios.

4.2 Generalized Eigenvector Solution of the Sum-Rate Optimization

From the receiver point of view array processing is more devoted to minimize the interference signal than to enhance the desired signal. In general, multipath effects and low desired signal power are mitigated via other kinds of diversity (time, fre-

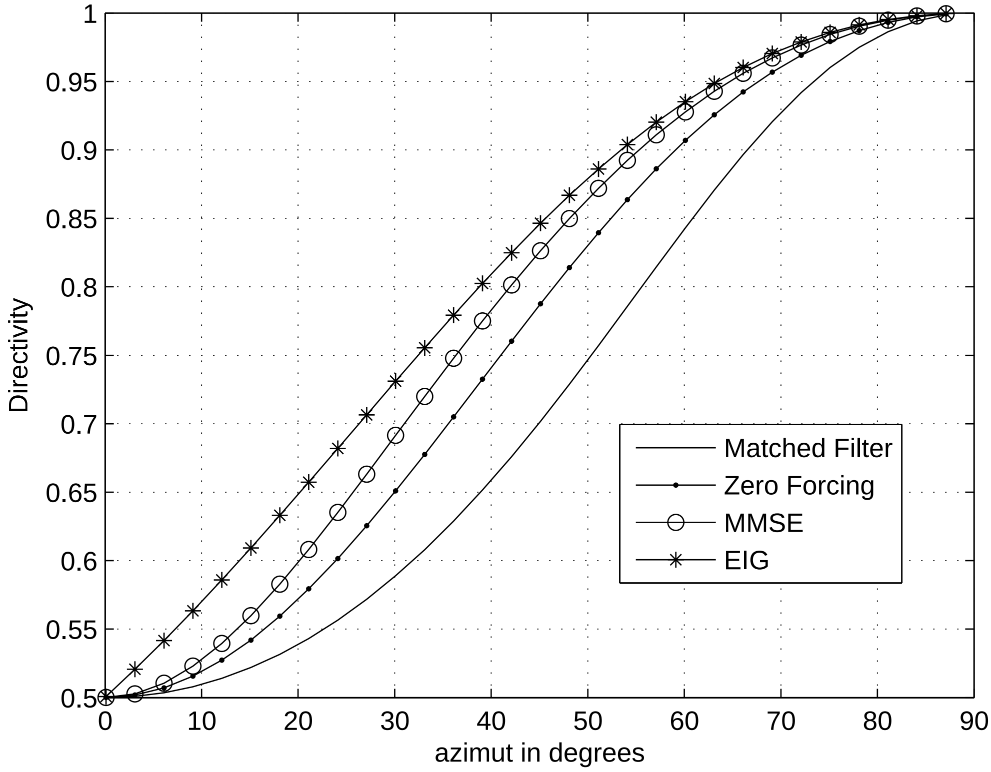

Figure 4.3: η versus interference angle of arrival, $P = 0dB$

quency and coding). In fact, for a reliable communication system the interference power level must be below the noise at the detection part. This is specially true when considering linear decoders whose performance is extremely degraded in presence of colored noise which is induced via the interference [25].

When the array processing is moved to the other communication side, the transmitter takes the role of interference reliever. This is of great importance in the MISO-IC since all users share time and frequency resources. Bearing this in mind, this section provides the sum-rate optimal beamformers considering that the interference power level is properly diminished by the beamformers.

Let us define the following power level values:

$$D_k = P|\mathbf{h}_{kk}^H \mathbf{b}_k|^2 \quad I_{jk} = P|\mathbf{h}_{jk}^H \mathbf{b}_j|^2 \tag{4.10}$$

which are the desired and the received interference by user j-th power levels. A performance metric of the overall network is the sum-rate defined as

$$R_S = \sum_{k=1}^{K} R_k. \tag{4.11}$$

Optimizing (4.11) w.r.t $\{\mathbf{b}_k\}_{k=1}^K$ is a difficult non-convex problem that have

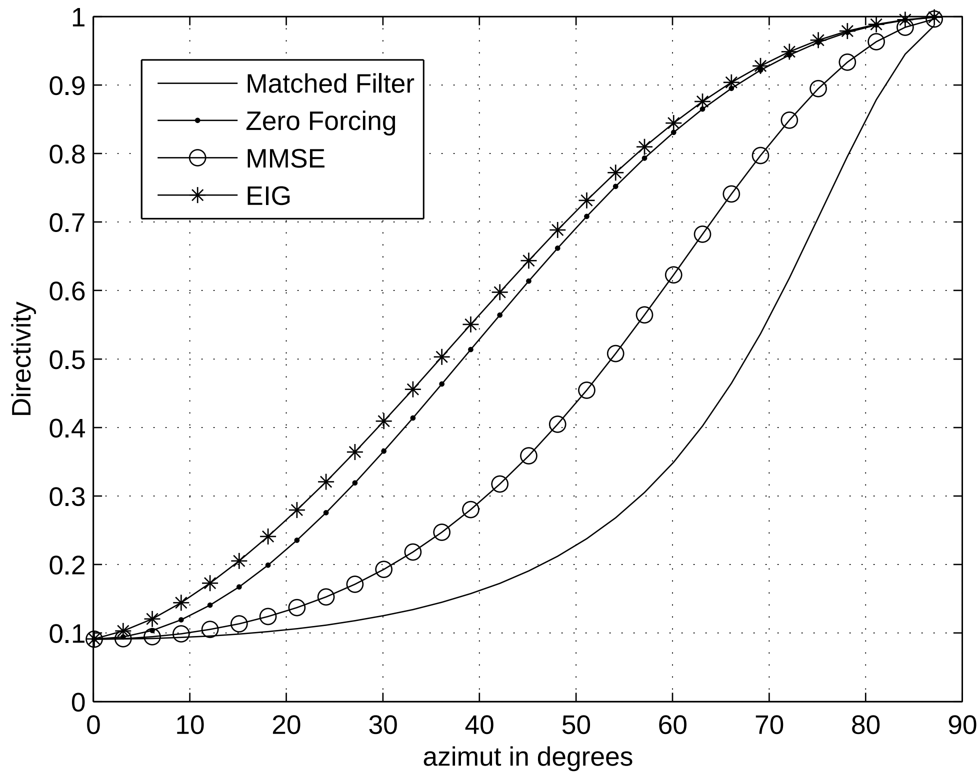

Figure 4.4: η versus interference angle of arrival, $P = 10dB$

gained a lot of attention recently. Relying on the optimal beamforming characterization of the previous chapter, different authors present a way of computing the beamforming parameters in order to obtain the sum rate optimal designs. Those works assume that there ir a central unit than can recollect all the channel vectors, optimize them and retransmit this information to the transmitter. Nota these methods have an extremelly high communication overhead that might severally decrease the spectral efficiency of the network.

On the other hand, it results impossible to obtain an optimal design wihtout any interchange of information between transmitters. Indeed, as the optimization problem is coupled, we cannot separatelly optimize the transmit beamformers although we consider special scenarios.

This the case of the designs presented in [28]. As it is described, when the number of users is 2 ($K = 2$), at low SNR scenario (i.e. $\frac{D_k}{\sigma^2} \to 0 \quad k = 1, 2$) the optimal transmit beamforming design is the matched filter

$$\mathbf{b}_k^{\mathrm{MF}} = \frac{\mathbf{h}_{kk}}{\|\mathbf{h}_{kk}\|} \quad k = 1, 2 \tag{4.12}$$

while at high SNR (i.e. $\frac{D_k}{\sigma^2} \to \infty \quad k = 1, 2$) the optimal transmit design is the zero forcing

$$\mathbf{b}_k^{\mathrm{ZF}} = \frac{\mathbf{P}_k \mathbf{h}_{kk}}{\|\mathbf{P}_k \mathbf{h}_{kk}\|} \quad k = 1, 2 \tag{4.13}$$

where $\mathbf{P}_k$ is the orthogonal projection matrix of the subspace spanned by the channel vectors of the non-intended receivers

$$\mathbf{P}_k \perp \mathrm{span}\{\mathbf{h}_{kj}\}_{k \neq j}^{K} \tag{4.14}$$

Note that this is a very intuitive result: with higher transmit power levels is used the interference must be extremelly supressed whereas in the low SNR regime, the transmit beamforming design must enhance the desired signal rather than supress interferences. Nevertheless, the derivation of this results is not straightforward as [28] shows.

When not only the desired signal power level is considered, but also the interference, the mentioned results do not hold. In other words, when the SINR is high in a two user scenario, this means that

$$\frac{D_k}{\sum_{j \neq k} I_{jk} + \sigma^2} \gg 1 \quad k = 1, 2 \quad j \neq k \tag{4.15}$$

and the optimal transmit design was derived in [7], becomes

$$\mathbf{b}_k^{\mathrm{VS}} = \alpha_k \left(P \mathbf{h}_{kj} \mathbf{h}_{kj}^H + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{h}_{kk} \quad k = 1, 2 \quad j \neq k \tag{4.16}$$

where α_k is set so that the beamformer has unit norm. This scheme was coined by 'R. Zakhour' *et. al* in [57] as 'Virtual SINR' design but it was also presented in [42, 7, 52, 3, 59, 2] in different contexts. Both in [59] and [7] Virtual SINR shows a good performance trade-off between the zero forcing and the matched filter designs. Moreover, an 'ad-hoc' extension for the K user case was presented in [57], which also appeared to achieve higher sum-rates than other designs.

We now proceed to present a novel beamforming design that also takes into account some desired and signal power levels in order to efficiently solve the sum-rate optimization. In contrast to the 'Virtual SINR' derivation, we focus our attention to the desired signal power level with respect to the interference, without taking into account the noise power level. This makes our solution more general as we will discuss in the next sections. For the sake of clarity, we first present the solution for the two user case and later for the more than two case.

4.2.1 $K = 2$

In order to obtain a decentralized sum-rate optimal beamformer for the two user MISO-IC we will assume the following.

As1) The SNR at each receiver is much larger than the Interference-to-Noise Ratio (INR). Mathematically,

$$\frac{D_k}{\sigma^2} \gg \frac{\sum_{j\neq k} I_{jk}}{\sigma^2} \quad k = 1, \dots, K \quad j \neq k \tag{4.17}$$

Note that this assumption does not only consider the desired signal power level, but also the interference level, in contrast to the zero forcing design (4.13) where it is assumed that the noise is negligible [28]. Moreover, it is important to remark that this assumption is realistic since after the beamforming effect it is desirable that the SNR is much larger than the INR in single user detection receivers.

Based on the aforementioned assumption, the next theorem follows.

Theorem 4.3. *The approximate sum-rate optimal beamformers when As1) holds and for the two user case are*

$$\left(P\mathbf{h}_{kk}\mathbf{h}_{kk}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k^{EIG} = \lambda_{max}\left(P\mathbf{h}_{kj}\mathbf{h}_{kj}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k^{EIG} \tag{4.18}$$

for $k = 1, 2$

Proof. We can rewrite (4.11) such as

$$R_S = \log\left(\prod_{k=1}^{K}\left(\frac{D_k + \sum_{j\neq k} I_{jk} + \sigma^2}{\sum_{j\neq k} I_{jk} + \sigma^2}\right)\right) \tag{4.19}$$

When As1) holds, (4.19) can be approximated by

$$\log\left(\prod_{k=1}^{K}\left(\frac{D_k + \sigma^2}{\sum_{j\neq k} I_{jk} + \sigma^2}\right)\right), \tag{4.20}$$

which for the two-user case can be decoupled in two different optimization problems with respect to each transmit beamformer

$$arg\max_{\mathbf{b}_k} \frac{P|\mathbf{h}_{kk}^H\mathbf{b}_k|^2 + \sigma^2}{P|\mathbf{h}_{kj}^H\mathbf{b}_k|^2 + \sigma^2}, \tag{4.21}$$

whose solution is the generalized eigenvector associated to the maximum generalized eigenvalue of the matrix pencil

$$\left(P\mathbf{h}_{kk}\mathbf{h}_{kk}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k = \lambda_{\max}\left(P\mathbf{h}_{kj}\mathbf{h}_{kj}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k \tag{4.22}$$

□

Although, the generalized eigenvector solution has been presented as optimal solution to the MISO wiretap channel [45], the presented beamformer appears to be novel in the MISO-IC scenario. We coined it as EIG beamforming.

Note that this beamformer appeared in the previous chapter as a solution of the directivity meausurament that we presented and evaluated. In this case, the α value is set to the noise power level but it is important to remark that in the directivity notion it represented the antenna power losses.

As for a solution for two users is insufficient for most of the wireless scenarios where at least three transmitters share time and frequency resources, in the next subsection we target the problem of more than one user optimization. In constrast to the presented derivation, when only applying As1), the problem continue to be coupled and; thus, more concrete assumptions are needed.

4.2.2 $K > 2$

For decoupling the optimization problem, we consider the next assumptions.

As2) The INR for each receiver k is low

$$\frac{\sum_{j \neq k} I_{jk}}{\sigma^2} \ll 1 \quad k = 1, \ldots, K \tag{4.23}$$

As3) The amount of interference that is created by each transmitter k and is experienced by the non-intended receivers with respect to the noise power level is low

$$\frac{\sum_{j \neq k} I_{kj}}{\sigma^2} \ll 1 \quad k = 1, \ldots, K \tag{4.24}$$

As there was the two user case, the already presented assumptions are easy to adapt to general wireless scenarios. Indeed, the amount of receive interference signal must be low in order to permit the communication. On the other hand, within the network, the transmitters are not meant to generate high levels of interference but to reduce them at least in several dBs.

Assuming the aforementioned signal power levels, we can derive the optimal beamformer for the more than one user case.

Theorem 4.4. *The optimal sum-rate design for the K-user case when As1)-As3) hold is*

$$\left(P\mathbf{h}_{kk}\mathbf{h}_{kk}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k^{EIG} = \lambda_{max}\left(P\sum_{j\neq k}^{K}\mathbf{h}_{kj}\mathbf{h}_{kj}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k^{EIG}$$

for $k = 1, \ldots, K$.

Proof. Manipulating (4.20) we arrive to

$$\sum_{k=1}^{K} \log\left(D_k + \sigma^2\right) - \sum_{k=1}^{K} \log\left(\sum_{j\neq k} I_{jk} + \sigma^2\right) \tag{4.25}$$

Since As2) holds, the second term can be approximated by

$$\sum_{k=1}^{K} \frac{\sum_{j\neq k} I_{jk}}{\sigma^2} + K\log\left(\sigma^2\right). \tag{4.26}$$

Now, the first term of expression (4.26) can be reformulated so that

$$\sum_{k=1}^{K} \frac{\sum_{j\neq k} I_{kj}}{\sigma^2} + K\log\left(\sigma^2\right). \tag{4.27}$$

Considering As3) we can finally write the sum-rate expression in (4.20) as

$$\log\left(\prod_{k=1}^{K}\left(\frac{D_k + \sigma^2}{\sum_{j\neq k} I_{kj} + \sigma^2}\right)\right), \tag{4.28}$$

which can be rewritten the following

$$\log\left(\prod_{k=1}^{K}\left(\frac{P|\mathbf{h}_{kk}^H \mathbf{b}_k|^2 + \sigma^2}{\sum_{j\neq k} P|\mathbf{h}_{kj}^H \mathbf{b}_k|^2 + \sigma^2}\right)\right). \tag{4.29}$$

As a consequence, the optimization of (4.29) can be done separately for each beamformer. Therefore, the optimal beamfomer for user k is

$$arg\max_{\mathbf{b}_k} \frac{P|\mathbf{h}_{kk}^H \mathbf{b}_k|^2 + \sigma^2}{\sum_{j\neq k} P|\mathbf{h}_{kj}^H \mathbf{b}_k|^2 + \sigma^2}, \tag{4.30}$$

whose solution is the generalized eigenvector associated to the maximum generalized eigenvalue of the matrix pencil

$$\left(P\mathbf{h}_{kk}\mathbf{h}_{kk}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k = \lambda_{\max}\left(P\sum_{j\neq k}^{K} \mathbf{h}_{kj}\mathbf{h}_{kj}^H + \sigma^2\mathbf{I}\right)\mathbf{b}_k \tag{4.31}$$

□

Again this beamforming design solves the directivity notion that we explained in the previous chapter. The K-user extension is naturally presented in the beamforming design so that while in the 2-user case there was only an interfere spatial signature, in this case there is a sum of them.

Due to its novelty in the transmit beamforming case, we must carefully undertand its properties. In other words, we must observe which the different in terms of array gain with respect to the other existing designs.

4.3 EIG beamforming Performance Evaluation

In this section we show that EIG beamforming presents an array gain closer to the zero forcing than other decentralized transmit beamformers. As we derived in the previous section, when not only the high SNR assumption is considered, but also its level with respect to the interference, the optimal transmit beamformer is no longer the zero forcing design but the EIG beamformer. Apparently, this new scheme does not completely block the amount of transmit interference in contrast to the zero forcing. In order to evaluate the novel design in front of the zero forcing, let us consider the following transmit beamformer evaluation meausure in an interfered scenario

$$\Phi(\mathbf{b}) = \frac{|\mathbf{b}_{ZF}^H \mathbf{b}|}{|\mathbf{b}_M^H \mathbf{b}|} \tag{4.32}$$

with

$$\mathbf{b}_{ZF} = \beta \left(\mathbf{I} - \mathbf{h}_i \mathbf{h}_i^H\right) \mathbf{h}_d \quad \mathbf{b}_M = \gamma \mathbf{h}_d, \tag{4.33}$$

where both β and γ are set so that $\mathbf{b}_{ZF}, \mathbf{b}_M$ are unit norm respectively. Vector $\mathbf{h}_d$ is the spatial signature of the intended receiver and $\mathbf{h}_i$ of the non-intended one. In the following, we assume $\sigma^2 = 1$ without lost of generality and by the sake of simplicity. Furthermore, we assume that there is only one interfered user although the same derivation can be done considering more users. In that case, the spatial signature $\mathbf{h}_i$ has to be changed to the spatial interfered subspace.

The metric (4.32) gives an idea of how a beamformer design is close to the zero forcing design. Indeed, for high values of $\Phi(\cdot)$ the beamformer is more close to the zero forcing design whereas for low values it is more close to the matched filter. Since zero forcing completely nulls the transmit interference, $\Phi(\cdot)$ shows the interference rejection capabilities with respect to the matched filter design (i.e. with respect to the array gain in the desired direction).

The next lemmas establish the expression for Φ in both cases: Virtual SINR and EIG beamformer. This metric will provide us a deeper understanding of the array gain of EIG beamforming.

Lemma 4.1. $\Phi(\cdot)$ evaluated with the EIG beamforming holds

$$\Phi(\mathbf{b}_{EIG}) = \frac{|\mathbf{b}_{ZF}^H \mathbf{h}_d| P}{(\lambda_{\max} - 1) \|\mathbf{h}_d\|} \tag{4.34}$$

Proof. We can rewrite (4.18) as

$$\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-1}\left(\mathbf{h}_d\mathbf{h}_d^H P + \mathbf{I}\right)\mathbf{b}_{EIG} = \lambda_{\max}\mathbf{b}_{EIG} \tag{4.35}$$

From the matrix inverse lemma we have

$$\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-1} = \left(\mathbf{I} - \frac{\mathbf{h}_i\mathbf{h}_i^H P}{1 + P\|\mathbf{h}_i\|^2}\right). \tag{4.36}$$

Due to this, the following equation holds

$$\mathbf{b}_{ZF}\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-1} = \mathbf{b}_{ZF} \tag{4.37}$$

Thus, (4.35) can be transformed into

$$\mathbf{b}_{ZF}^H\mathbf{h}_d\mathbf{h}_d^H\mathbf{b}_{EIG}P = (\lambda_{\max} - 1)\mathbf{b}_{ZF}^H\mathbf{b}_{EIG}, \tag{4.38}$$

and by considering the definition in (4.33), it is easy to arrive to (4.34). □

Lemma 4.2. $\Phi(\cdot)$ evaluated with the 'Virtual SINR' beamforming holds

$$\Phi(\mathbf{b}_{VS}) = \frac{|\mathbf{b}_{ZF}^H\mathbf{h}_d|}{\mathbf{h}_d^H\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-1}\mathbf{h}_d\|\mathbf{h}_d\|} \tag{4.39}$$

Proof. It is trivial considering (4.37) □

Considering this meausurement which gives a notion of the interference rejection capabilities in front of the enhancement of the desired signal power level, we now provide to compare the Virtual SINR and EIG beamforming designs.

Theorem 4.5.

$$\Phi(\mathbf{b}_{EIG}) > \Phi(\mathbf{b}_{VS}) \tag{4.40}$$

Proof. Operating (4.40) we want to show

$$\frac{P}{\lambda_{\max} - 1} > \frac{1}{\mathbf{h}_d^H\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-1}\mathbf{h}_d} \tag{4.41}$$

From its definition we know that for EIG beamforming we have

$$\lambda_{\max} = \frac{\mathbf{b}_{EIG}^H\left(\mathbf{h}_d\mathbf{h}_d^H P + \mathbf{I}\right)\mathbf{b}_{EIG}}{\mathbf{b}_{EIG}^H\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)\mathbf{b}_{EIG}} \tag{4.42}$$

Since $\mathbf{b}_{EIG}$ has unit norm, we have that

$$\frac{P}{\lambda_{\max} - 1} \geq \frac{\mathbf{b}_{EIG}^H\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)\mathbf{b}_{EIG}}{|\mathbf{b}_{EIG}^H\mathbf{h}_d|^2} \tag{4.43}$$

Considering the Cauchy-Schwarz inequalitie, i.e.

$$\|\mathbf{u}\|^2\|\mathbf{v}\|^2 \geq \|\mathbf{u}^H\mathbf{v}\|^2 \tag{4.44}$$

and defining

$$\mathbf{u} = \left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{\frac{1}{2}} \mathbf{b}_{EIG} \tag{4.45}$$

$$\mathbf{v} = \left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-\frac{1}{2}} \mathbf{h}_d \tag{4.46}$$

then it is easy to show that

$$\frac{\mathbf{b}_{EIG}^H\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)\mathbf{b}_{EIG}}{|\mathbf{b}_{EIG}^H\mathbf{h}_d|^2} \geq \frac{1}{\mathbf{h}_d^H\left(\mathbf{h}_i\mathbf{h}_i^H P + \mathbf{I}\right)^{-1}\mathbf{h}_d} \tag{4.47}$$

and (4.41) follows. □

With this theorem, we can state that EIG beamforming has better rejection capabilities than the Virtual SINR. As we will see in the numerical evaluation section, this is specially true when the transmit power is low. This effect is extremely important for some scenarios that EIG beamforming shows a better performance with respect to the other decentralized designs.

Recalling the previous chapter, where the achievable rates where deeply studied, it is compulsory to check whether our beamforming proposal fits with those optimal designs. Indeed, as stated in [37] any point of the Pareto Rate region of the K user interference channel can be achieve if all transmitters design their beamformers so that

$$\mathbf{b}_k = \mathbf{v}_{max}\left(\sum_{l=1}^{K}\delta_{kl}e_l\mathbf{h}_{kl}\mathbf{h}_{kl}^H\right) \tag{4.48}$$

where

$$\sum_{j=1}^{K}\delta_{kj} = 1 \quad \delta_{kj} \in [0,1] \quad j = 1,\dots K \tag{4.49}$$

$$e_l = -1 \quad \text{if} \quad l \neq k \quad \text{otherwise} \quad e_k = 1. \tag{4.50}$$

In the next proposition we will see how EIG beamforming can be casted as the set of beamformers described in (4.48).

Proposition 4.1 EIG beamforming can attain the Pareto rate region of the MISO IC.

Proof. Operating EIG beamforming expression, it results that

$$\left(P\mathbf{h}_{kk}\mathbf{h}_{kk}^{H} - P\lambda_{\max}\sum_{j\neq i}\mathbf{h}_{jk}\mathbf{h}_{jk}^{H}\right)\mathbf{b}_{k}^{EIG} = \sigma^{2}(\lambda_{\max}-1)\mathbf{b}_{k}^{EIG} \tag{4.51}$$

which takes the form of (4.48). As a consequence, a rate Pareto point can be achieved by EIG beamformer. □

With this we can observe the development of the EIG beamforming as an set of the λ_k values. Indeed, we focus on a beamformer that can be computed in a decentralized fashion and, as it is easy to observe, the identification of the λ_k lead to constants that can be computed by each beamformer separately. It is important to remark that this beamformer might not achieve the Pareto rate region but, when the assumptions hold, it achieves the sum rate and; thus, the Pareto rate frontier.

4.4 Numerical Simulations

We first evaluate $\Phi(\cdot)$ for the Virtual SINR and the EIG beamformer versus the transmitted power given two different Gaussian channel realizations, namely desired and interference, with unit norm and when 4 antennas are considered. In fig. 4.5 it can be seen that EIG gives a better Φ than the Virtual SINR beamformer. It can be observed that the difference becomes higher at low power transmission.

In [51] we establish that for the symmetric case EIG performs similar to Virtual SINR we now focus on evaluating the asymmetric case. Therefore, we parametrize the ratio between the direct channel gains so that

$$\beta = \frac{||\mathbf{h}_{11}||}{||\mathbf{h}_{22}||} \tag{4.52}$$

and the interfered channel gains are set to one as well as $||\mathbf{h}_{22}|| = 1$. All numerical results are obtained via Monte Carlo simulations with fading Rayleigh channel model over 10000 realizations. Moreover, we assume that $\sigma^2 = 1$.

Fig. 4.6 shows the sum-rate versus the SNR for the 2 antenna case when $\beta = 10dB$. In this scenario it can be seen that the zero forcer is the best solution at high SNR. For this scenario, EIG can increase the sum-rate in 0.3 bits per channel use at 2 dB of SNR over the rest of existing techniques. Note that under this range both SINR and SNR are high but EIG performs better than Virtual SINR. We also evaluate our proposal for the three user cases when also the channels are asymmetric $\beta = \frac{||\mathbf{h}_{11}||}{||\mathbf{h}_{22}||} = \frac{||\mathbf{h}_{11}||}{||\mathbf{h}_{33}||} = 10dB$ and when the transmitters are equipped with 3 antennas. For this case, we can also observe in Fig. 4.7 that EIG outperforms the decentralized transmit beamforming designs in a determined range of SNR.

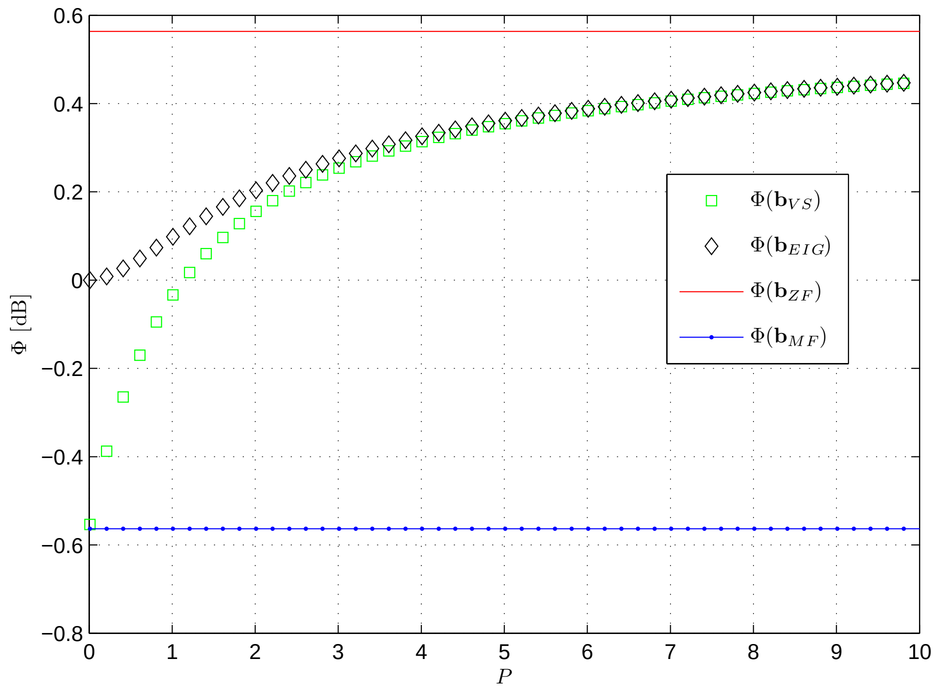

Figure 4.5: $\Phi(\cdot)$ is evaluated for EIG beamforming and Virtual SINR for a range of transmit power.

In order to understand this effect we depict in Fig. 4.8 and 4.9 the INR values of both users in the 2-user case scenario. It is clear that for user 1 the amount of interference is much lower. Indeed, for this case EIG can attenuate the interference more than 20 dB with respect to Virtual SINR. And this is the reason of the superior performance of EIG: the transmit beamformer of user 2 is more devoted to reduce the interference to user 1 rather than to enhance the desired one. Intuitively, since the direct channel gain is very degraded the sum-rate optimal design for the beamformer of user 2 is to cancel interference. As a general statement, *in a MISO interference network, the transmitters whose direct channel gain is very low, have to focus on reducing the interference to the non-intended receivers rather than on maximizing the direct link gain.* As we have shown in the previous section, EIG behaves very close to the zero forcing technique and, thus, has better interference rejection capabilities. It is important to mention that this operating point is not fair in terms of rate (i.e. provides more rate to one user than the other) although it presents higher sum-rate.

As an extra point to motivate the use of EIG beamforming in spectrum sharing networks, we present a receiver beamforming design that has the same structure as EIG. Indeed, the our proposal design can be viewed as a reciprocal beamforming from a receive one that takes into account an AGC. The derivation and the receiver structure is presented for the sake of completeness.

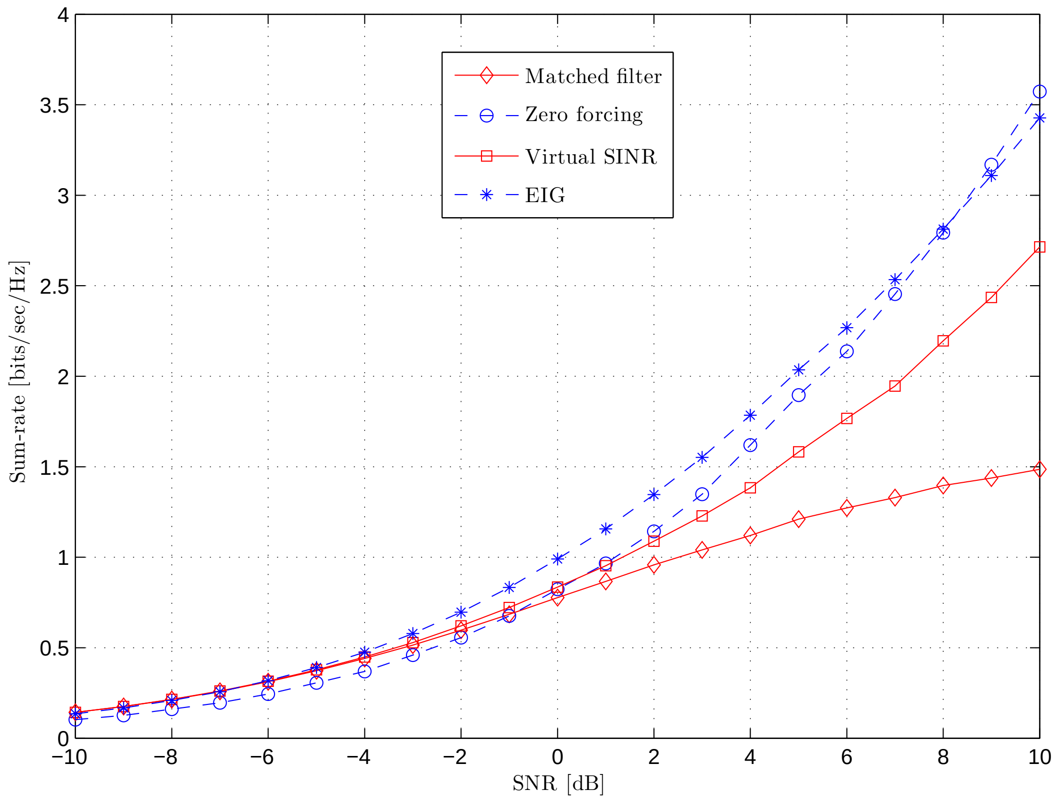

Figure 4.6: Sum-rate for 2-user MISO IC with $N = 2$ antennas, $\beta = 10dB$.

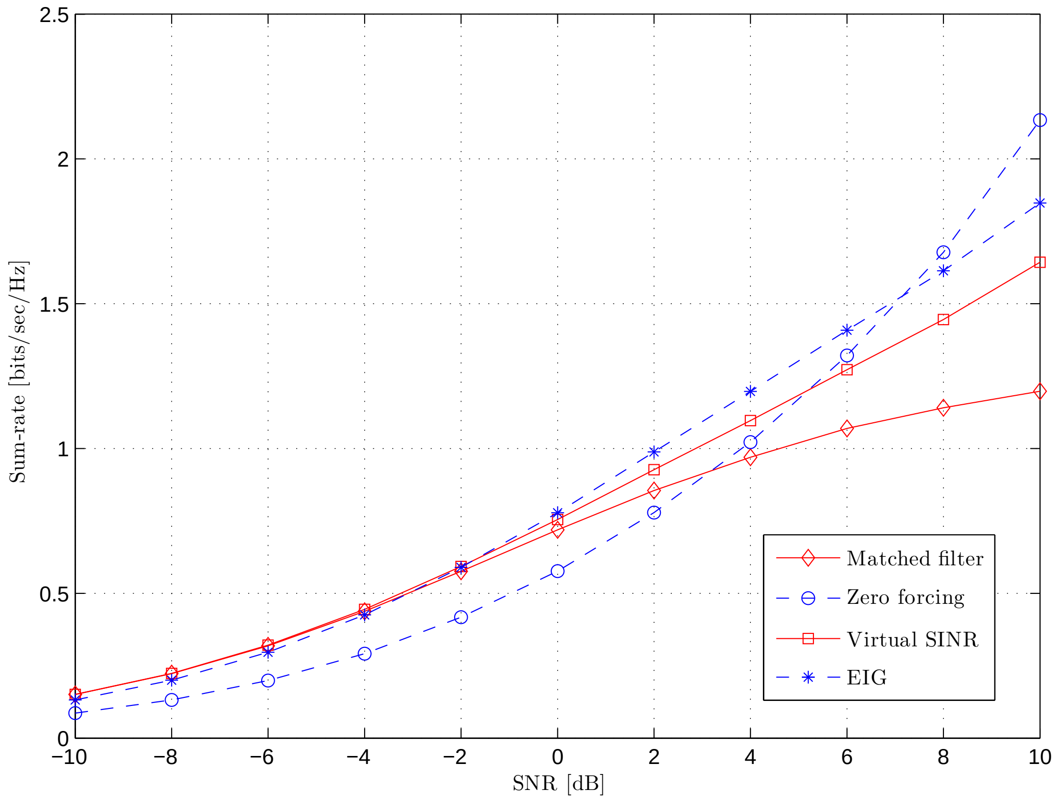

Figure 4.7: Sum for the 3-user case and $N = 3$ antennas.

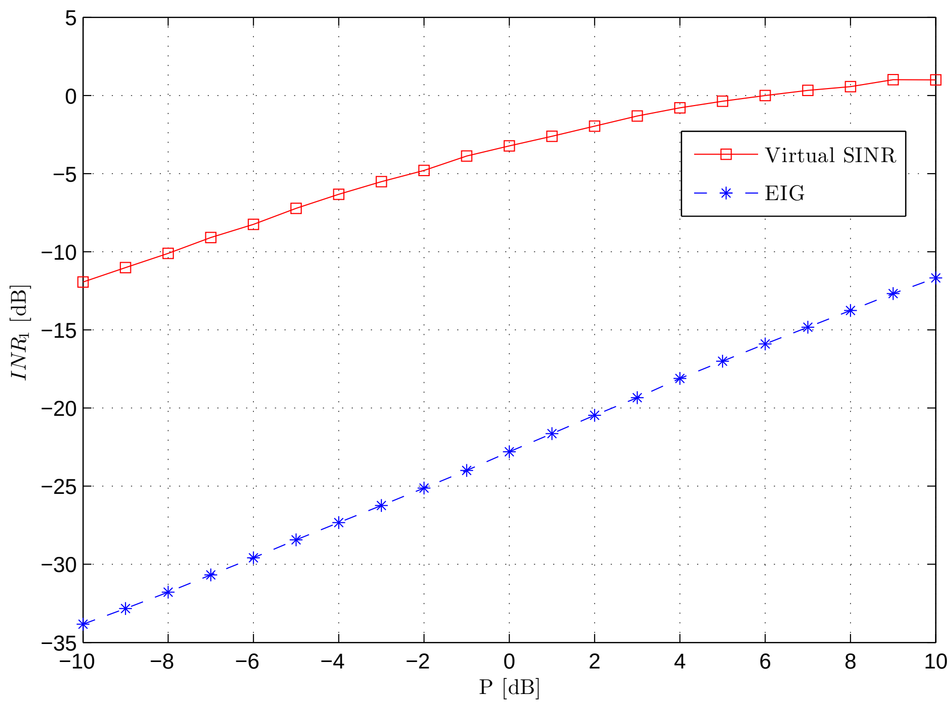

Figure 4.8: INR of user 1 for 2-user MISO IC with $N = 2$ antennas, $\beta = 10dB$

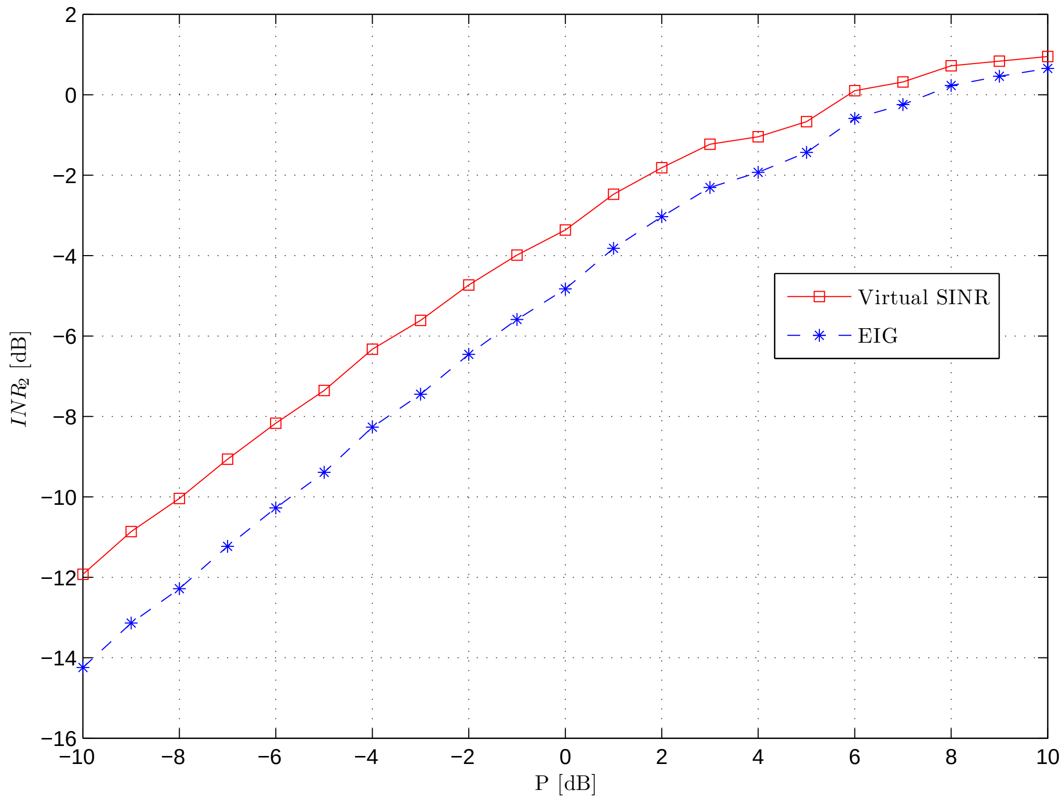

Figure 4.9: INR of user 2 for 2-user MISO IC with $N = 2$ antennas, $\beta = 10dB$

4.5 Receive Power Beamforming with AGC

4.5.1 System Model

In a wireless communication system, the baseband received signal by an array of N antennas at time instant n can be expressed as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{i}(n) + \mathbf{w}(n) \tag{4.53}$$

where $\mathbf{H} \in \mathbb{C}^{N \times L}$ encompasses the equivalent channels for the L received snapshots at each time arrival

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_L] \tag{4.54}$$

where $\mathbf{h}_l, \quad l = 1, \ldots, L$ is the spatial signature of the channel at time instant l. $\mathbf{x}(n) \in \mathbb{C}^{L \times 1}$ denotes the transmitted signal. $\mathbf{i}(n) \in \mathbb{C}^{N \times 1}$ denotes the received interference and we assume that $\mathbb{E}[\mathbf{i}^H(n)\mathbf{x}(n)] = 0, \forall n$. $\mathbf{w}(n) \in \mathbb{C}^{N \times 1}$ denotes the additive uncorrelated white circular gaussian noise with σ^2 variance.

The resulting signal after the spatial processing becomes

$$r(n) = \mathbf{a}^H \mathbf{y}(n) \tag{4.55}$$

where $\mathbf{a} \in \mathbb{C}^{N \times 1}$ denotes the receiver beamformer. As we will see in the next subsection, the role of $\mathbf{a}$ is to reject undesired signals (either late arrivals or interferences).

4.5.2 Receive Beamforming with AGC constraints

Line-of-Sight Scenario

Most of the adaptive beamforming techniques rely on a reference signal that framed together with the desired one allows the proper processing of the received snapshots. It can be stated that the acquisition, synchronization and the full and reliable regeneration of the reference $d(n)$ entails the most difficult engineering part of the array processing at the receiver. Here we will focus on an aspect that becomes relevant for the beamforming procedure, which is the automatic gain control or AGC. It is well known that AGC is crucial for detection of constellations in communications that are loaded with more than two bits. In fact, the dynamic range control required for the baseband part of the receiver is quite demanding and produces severe degradation in performance when it is not properly set. Next, the simplicity of a TRB is described in order to face the AGC problem in the easiest case.

Initially we consider a scenario without multipath $L = 1$. For this case, we have

$$\mathbf{y}(n) = \mathbf{h}_1 d(n) + \mathbf{i}(n) + \mathbf{w}(n) \tag{4.56}$$

where $d(n)$ is the reference signal, which is known at the receiver and usually regenerated from the incoming reference. Given this reference, the design of a narrowband beamformer $\mathbf{a}$ is done based on the minimization of

$$\min_{\mathbf{a}} \mathbb{E}[|\mathbf{a}^H \mathbf{y}(n) - d(n)|^2] \tag{4.57}$$

The reference $d(n)$ is assumed with normalized power equal to one. If the incoming signal has a level substantially different from $d(n)$, the weigth vector has to scale accordingly and it can create problems to the control of the dynamic range. It would be much more desirable to scale the reference $d(n)$ such that the dynamic range at the beamforming output would remain fix. This is the reason for the importance of including a jointly design AGC and beamforming. With this joint design the objective is not longer (4.57) but

$$\min_{\mathbf{a},\alpha} \mathbb{E}[|\mathbf{a}^H \mathbf{y}(n) - \alpha d(n)|^2] \tag{4.58}$$

which can be rewriten

$$\min_{\mathbf{a},\alpha} \mathbb{E}[|\mathbf{a}^H \mathbf{h}_1 d(n) - \alpha d(n)|^2] + \mathbf{a}^H \left(\mathbf{R} - \mathbf{h}_1 \mathbf{h}_1^H\right) \mathbf{a} \tag{4.59}$$

Since the objective admits the trivial solution of setting both beamformer and AGC equal to zero, it is clear that some additional constraint is needed. Depending on this constraint, the resulting beamformer may change, in some cases, dramatically. As the aim is to remove undesired fluctuations on the array output, the constraint is set on terms of the beamformer's output. The AGC is left unconstrained and free to minimize the objective (4.58)

$$\alpha = \mathbf{h}_1^H \mathbf{a} \tag{4.60}$$

where $\mathbf{h}_1$ can be estimated from data as

$$\hat{\mathbf{h}_1} = \mathbb{E}[\mathbf{y}(n) d^*(n)] \tag{4.61}$$

After inserting the optimum AGC (4.60) in the objective (4.58), we obtain

$$\min_{\mathbf{a}} \mathbf{a}^H \left(\mathbf{R} - \mathbf{h}_1 \mathbf{h}_1^H\right) \mathbf{a} \tag{4.62}$$

where $\mathbf{R} = \mathbb{E}[\mathbf{y}(n)\mathbf{y}(n)^H]$, and therefore

$$\mathbf{R} = \mathbf{R}_D + \mathbf{R}_I + \sigma^2 \mathbf{I} \tag{4.63}$$

where $\mathbf{R}_D = \mathbf{h}_1 \mathbf{h}_1^H$, $\mathbf{R}_I = \mathbb{E}[\mathbf{i}(n)\mathbf{i}(n)^H]$. Note that the beamformer depends solely on the front-end noise, the estimation quality of $\hat{\mathbf{h}}_1$ and the interference. For the design of its constraint there are several possible choices:

1. Fix the noise front-end power: $\mathbf{a}^H\mathbf{a} = 1$
2. Fix the response to the desired signal: $\mathbf{a}^H\mathbf{R}_D\mathbf{a} = 1$
3. Fix the response to the desired signal plus front-end noise: $\mathbf{a}^H\left(\mathbf{R}_D + \sigma^2\mathbf{I}\right)\mathbf{a} = 1$

The first constraint does not have much practical interest, since it does not take into account the desired signal. The solution for the beamformer is the eigenvector that is associated to the minimum eigenvalue of the matrix

$$\mathbf{a}_{\mathsf{MIN}} = \lambda_{\mathsf{min}}\left(\mathbf{R}_I + \sigma^2\mathbf{I}\right) \tag{4.64}$$

we coined as MIN.

The second constraint ensures a proper level of the desired signal and only in the case that the desired autocorrelation matrix is rank one it coincides with the MMSE solution, this is

$$\mathbf{a}_{\mathsf{MMSE}} = \left(\mathbf{R}_I + \sigma^2\mathbf{I}\right)^{-1}\mathbf{h}_1 \tag{4.65}$$

The third constraint presents a refinement versus case 2 because it takes also into account the front-end noise when setting the array output power level at its real value. The beamformer solution is the maximum generalized eigenvector of the matrix pencil $(\sigma^2\mathbf{I} + \mathbf{R}_D, \sigma^2\mathbf{I} + \mathbf{R}_I)$. In our case, it can be reducted to

$$\mathbf{a}_{\mathsf{EIG}} = \lambda_{\mathsf{max}}\left(\left(\sigma^2\mathbf{I} + \mathbf{R}_I\right)^{-1}\left(\sigma^2\mathbf{I} + \mathbf{R}_D\right)\right) \tag{4.66}$$

We will called it EIG and it is the reciprocal design of the one derived from the new notion of directivity and the sum-rate optimal one when some interference power level assumptions are considered.

ISI and coherent multipath scenarios

We now describe the MDIR receiver (Figure 4.10). This receiver was reported several years ago by the authors [27] in order to cope with the joint design of beamforming and the DIR of the sequence detector for frequency selective communications channels.

Assuming that the propagation suffers from selective fading due to multipath, the received snapshot will be formed as it is shown in (4.67), where, without loss of generality, we assume that there is a LOS component, together with early arrivals, which are produced by reflections usually close to the transmitter or receiver site, and

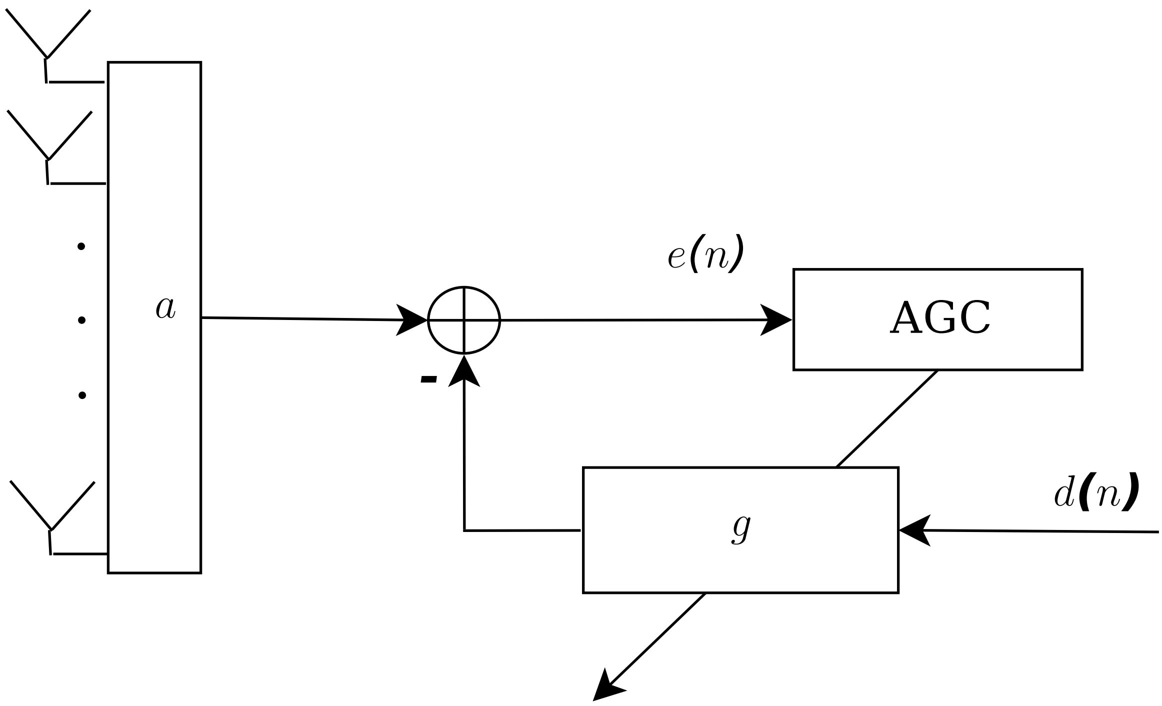

Figure 4.10: MDIR Architecture

late arrivals. Our interest is that early arrivals contain a significant amount of energy that may help in the detection. On the other hand, late arrivals are not desirable for the receiver because they present low energy together with low statistical stability. In any case, our interest is mainly to have the possibility of managing multipath, i.e. selecting only the LOS component, or only the early arrivals or all together for the detector. Let us assume that we are interested in the LOS and early arrivals, which is most useful for a Viterbi detector.

$$\mathbf{y}(n) = \mathbf{H}\begin{bmatrix} d(n) \\ \mathbf{d}_{\mathsf{early}}(n) \\ \mathbf{d}_{\mathsf{late}}(n) \end{bmatrix} + \mathbf{i}(n) + \mathbf{w}(n) \tag{4.67}$$

where $\mathbf{d}_{\mathsf{early}}(n)$ is a vector that contains the early arrivals and $\mathbf{d}_{\mathsf{late}}(n)$ is a vector that contains the late arrivals. The received covariance matrix becomes

$$\mathbf{R} = \mathbf{H}\mathbf{H}^H + \mathbf{R}_I + \sigma^2\mathbf{I} \tag{4.68}$$

Let us assume that we are interested in the LOS and early arrivals, which is most useful for a Viterbi detector. Notice that now is not anymore a sequence but a vector of sequences and, therefore, the temporal processing is not a scalar α but a vector $\mathbf{g}$ which is coined as Desired Impulse Response (DIR).

Considering the constraint 2, the design of the beamformer together with the

DIR is

$$\mathbf{g} = \mathbf{H}_d \mathbf{a} \tag{4.69}$$

$$\mathbf{a} = \lambda_{\max}\left(\left(\mathbf{R} - \mathbf{H}_d \mathbf{H}_d^H\right)^{-1} \left(\mathbf{H}_d \mathbf{H}_d^H\right)\right) \tag{4.70}$$

where $\mathbf{H}_d \in \mathbb{C}^{N\times(L_e+1)}$, being L_e the total number of early arrivals, retains the spatial signature of both the LOS arrival and the early ones. Note that this design attenuates late arrivals as well as the intereference. The desired energy associated with LOS and early arrivals is left for the DIR output.

Figure 4.11 shows the array factor of the resulting beamforming design using the third constraint. The scenario consists on a desired QAM (Quadrature Amplitud Modulated) signal that is sampled at the symbol rate. The arrival set is composed of 3 paths: the LOS, early and late arrival. The DOAs of the arrivals are 0, 20 and 40 dB respectively. The DIR assumes, as stated before, only two paths, in consequence the third arrival is considered late and causes interference if it is not properly removed or attenuated by the beamformer. The figure clearly shows that the beamformer performs its assigned job. Note that late arrivals use to be unstable, in consequence they do not use to be included in the DIR. In case the spatial processing cannot remove them, they form part of the residual ISI term. As an example, GSM receivers set the length of the DIR to a maximum of 5 consecutive arrivals.

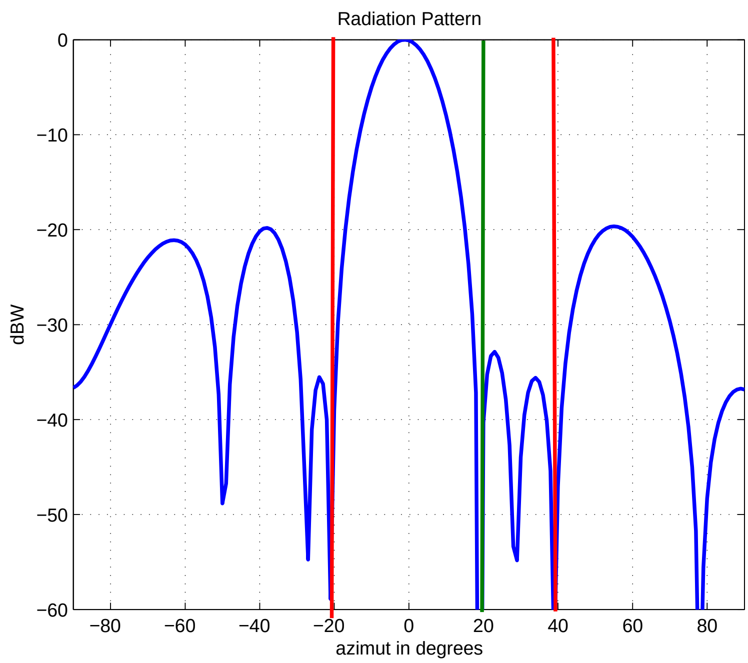

Figure 4.11: Array factor eliminating the late arrival

In Figure 4.12, the length of the DIR has ben extended to include the late arrival.

As it can be seen, the resulting beamformer includes this arrival at its output as desired signal.

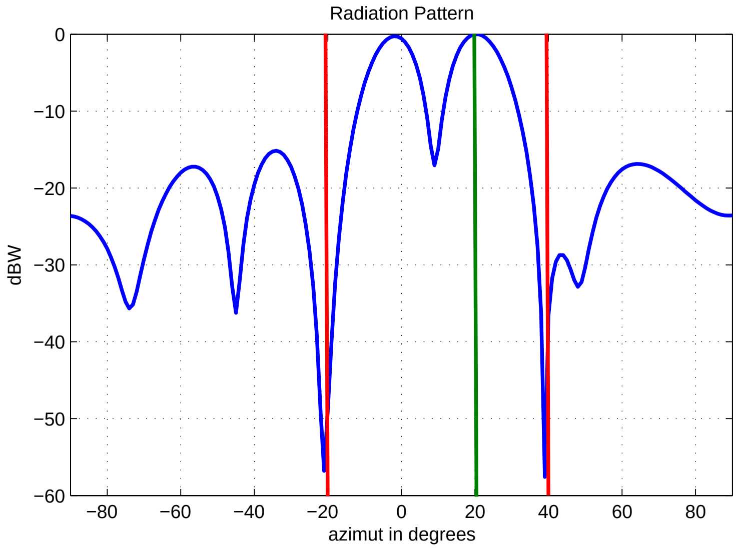

Figure 4.12: Array factor capturing the late arrival

It is important to note that the beamformer, by itself, is not the proper receiver since it delivers at its output the desired signal with intersymbol interference; thus it is the joint work of the beamformer and the DIR that forms the sequence detector that is able to deliver proper estimates of the transmitted symbols. In other words, sometimes these beamformers derived for case 2 and 3 are named as maximizers of the SINR, but the resulting SINR is not valid until the DIR is used to resolve the transmitted symbols. An additional remark is that, the alternative to the search in the sequence detector, sometimes called as Viterbi detector, can be avoided by a decision feedback equalizer DFE, at the expense of error propagation. Nevertheless, the use of DFE instead of the DIR receiver reports good results on the medium high SNR scenarios, or alternatively, on those systems working on row BER below 10^{-3}.

Finally, the AGC of 1 reduces the beamformer to the minimum eigenvector of the interference matrix $\mathbf{R}_I$. As mentioned before, this beamformer does not guarantee a proper level of the desired at the output and, even worse, it may promote severe attenuation of the desired arrivals.

As we have seen, AGC plays an important role in the receiver beamforming design. Indeed, different solutions are obtained in case different restrictions are imposed. As we have seen, these AGC restrictions they are also translated to different transmit beamforming designs. Concretelly, when total receive power is constraint, it leads to the EIG beamforming design.

4.6 Summary

We have presented a novel transmit beamformer for spectrum sharing communications. This novel beamformer is motivated via three reasonings: the sum-rate optimization, a notion of directivity for spectrum sharing systems and the reciprocal version of a receive beamforming. Our proposal has been carefully analysed and evaluated through numerical simulations. Therefore, EIG is meant to become a key design for next generation wireless systems.

In those future spectrum sharing systems interference will play an important role, specially when considering the coexistence between different systems and different spectrum management regulations. Next chapter introduces an idea to improve these systems which relays on the fact that the total amount of receive power must remain constraint in order to promote coexistence between different systems and to increase the spectral efficiency.

Appendix 4.A Transmit Beamforming that optimizes η_1, η_2 and η

The efficiency parameter η_1 can be rewritten as a Rayleigh quotient

$$\eta_1 = \frac{\mathbf{w}^H \mathbf{A}_1 \mathbf{w}}{\mathbf{w}^H \mathbf{B}_1 \mathbf{w}} \tag{4.71}$$

where

$$\mathbf{A}_1 = \mathbf{s}_{\text{user}} \mathbf{s}_{\text{user}}^H \tag{4.72}$$

$$\mathbf{B}_1 = \sum_{q=1}^{Q} \mathbf{s}_q^i (\mathbf{s}_q^i)^H + \alpha \mathbf{I} \tag{4.73}$$

Although the optimization of (4.71) with respect to $\mathbf{w}$ is a non-convex problem, it is well known that its maximum value is reached when $\mathbf{w}$ takes the form of

$$\mathbf{A}_1 \mathbf{w} = \lambda_{\max} \mathbf{B}_1 \mathbf{w} \tag{4.74}$$

for the case of under study, we have

$$\mathbf{s}_{\text{user}} c = \lambda_{\max} \left(\sum_{q=1}^{Q} \mathbf{s}_q^i (\mathbf{s}_q^i)^H + \alpha \mathbf{I} \right) \mathbf{w} \tag{4.75}$$

where

$$c = \mathbf{s}_{\text{user}}^H \mathbf{w} \tag{4.76}$$

It is clear that (4.75) is equivalent to the design of the coined virtual SINR. Note that the optimization of (4.71) as for any Rayleigh quotient, is independent of the norm of vector $\mathbf{w}$. When considering η_2, we get an equivalent result, but for this case we have a different Rayleigh quotient

$$\mathbf{A}_2 = \mathbf{s}_{\text{user}}\mathbf{s}_{\text{user}}^H - \left(\sum_{q=1}^{Q} \mathbf{s}_q^i(\mathbf{s}_q^i)^H + \alpha\mathbf{I}\right) \tag{4.77}$$

$$\mathbf{B}_2 = \alpha\mathbf{I} \tag{4.78}$$

where it is clear that the final design is equivalent to the virtual SINR one. Similar reasoning can be used for obtaining EIG beamforming scheme from η.

Chapter 5

Transmit Beamforming with Receive Power Constraints

Most current data traffic is delivered to the final user via short or medium range open spectrum systems. These systems, which work in the Industrial Scientific and Medical (ISM) band, have been proliferating in the recent years so that off-the-shelf technological equipment can be integrated with a very low cost overhead. Note that the optimization of those systems are indeed the target of this dissertation. Not only its current optimization but also its extension will be studied in this chapter.

As a matter of fact, the cornerstone of the fast spread of open spectrum communication is regulation. Indeed, the potential of spectrum sharing systems relies on its 'free' conception, as any transmitter can send information within a maximum radiated power. Although limiting the transmit power avoids long range communications, it provides a better coexistence between different systems. Therefore, for targeting wider area spectrum sharing wireless network, a new spectrum management policy is needed as well as new regulation rules in order to ensure the coexistence of different adjacent networks.

In order to solve this problem, time ago it was proposed the TAS licenses, first presented in [13], provide a complete open spectrum management system that allows to put on the telecom market the cognitive radio technology. Citing the original manuscript on TAS, it proposes a license so that the owner has

- 'the exclusive right to originate radiation subject to the constraint that the field strength achieved by this radiation does not exceed a specified limit, expressed in volts per meter (X V/m) at any point outside his area'
- 'the right to be free, above the same field strength (X V/m), from radiation originating in any other area'

In other words, this regulation technique assigns to a specific operator the right of transmitting in a given frequency for a certain portion of time within some geographical limits. Exploiting not only frequency, but also time and space in a regulatory fashion is the basic strategy for interference management. In this way, different wireless communication networks can coexist in neighbouring geographical areas, yet maintaining a low inter-network interference. The benefits of these spectrum management systems are discussed in next section, which argues why the spatial spectrum constraints can be approximated by the received power constraints. Note that, although the SINR might be moderate, the received power can be extremely high. The current systems add an AGC just after the transmit antenna such that the SINR is adjusted [54] and, generally, optimal transmit designs only consider transmit power constraint [12, 49].

The pioneering M. Gastpar's works [17, 18, 19] proposes TAS and as the regulator is only interested in the radiated power that can be properly measured by the received power, the paper studies how the capacity of the system is modified when only received power constraints are taken into account. Thus, changing the existing focus on the transmit power so far. Besides, a further research within the relay amplify-and-forward scenario can be found in [4].

This chapter focuses on the problem of designing the transmit beamformer and power control for an interference network when each receiver constrains the total amount of received power. This constraint is specified by the technology provider, who produces, for the network operators, receivers that are under standard qualifications and regulation. Note also that being an interference network entails that neither the transmitters nor the receivers share information, which contrasts with the well-known downlink channel designs [6, 5, 43, 22].

Firstly, we obtain the achievable rate region (i.e. the Pareto rate region) given a channel gain matrix and a set of received power constraints. This is derived via multicriteria optimization theory, which provides the opportunity to generalize previous results of the achievable rates of interference networks [9]. Among the different power Pareto optima we are interested in the one that fulfills the received power masks with equality. For the two-user case we show that this operating point achieves maximum sum-rate, when there is enough available transmit power.

Next, we propose a beamformer design that allows to reach the desired operating point. As the problem of obtaining the maximum sum-rate beamformer under received and also available transmit power is highly coupled and non-convex, this work takes into account some practical assumptions. As a result we provide an engineering solution to a complex problem and obtain a decentralized beamformer with a closed form expression. This beamformer is compared against other well-known transmit beamformers.

The rest of the chapter is organized as follows. Section II provides insights into the received power restrictions. In section III the system model is presented. In section IV the optimal transmit power policy is identified, the achievable rates are derived and an iterative method is presented. Section V shows different transmit beamforming designs which preserve a decentralized fashion, when compared to them the proposed novel beamformer fulfills both the transmit and the received power constraints. Section VI shows the numerical simulations of both beamforming and power control, jointly. Section VII concludes.

5.1 Practical Considerations on Power Constraints

Traditionally, most of the wireless designs, both for maximizing rate or Signal-to-Noise Ratio (SNR), use to be done under a maximum available power, P_{max}. Nevertheless, this equipment parameter is only important for battery powered transmitters, whenever it is assumed that the major demand from battery is the transmit subsystem. In other cases, the bound on the available power is justified in terms of the maximum power that is supported by the radio-frequency amplifier.

Meanwhile the use of the available power constraint has some sense on single antenna transmitters, the use of antenna arrays precludes a clear relationship between global available power and the average or peak power per amplifier. The conventional power restriction P_{max} is formulated as an average constraint across the antennas, which results very attractive from the mathematical point of view, but it is unrealistic in practice [39, 56].

The important and restricting parameter is the radiated power density P_{RA}[Watts/m^2], which entails the antenna gain, the directivity and the coverage. It also describes the degree of contamination of the radio-spectrum and is key for a proper interference management. At a specific location of a receiver, power density P_{RA} translates into specific received power. Therefore, along the chapter we refer to the regulation bound in terms of received power. In general, power of overlapping signals at each location should not exceed the maximum power flux density (Watts/Hz) allowed by radio regulations, which translates into a received power when it is evaluated in the working bandwidth of the receiver.

We comment that there exist works in the literature as [48] that consider the joint optimization of the mutual information with both constraints (i.e. radiated and available power) and different results are obtained with respect to the works that only consider available power. However, the precoder design when the radiated power is considered relays on the full knowledge of the coupling matrix of the antenna array and it results very difficult to obtain in practice and to control at manufacturing. Also, the joint optimization is a difficult mathematical problem. The use of received power constraints instead of the radiated ones overcomes these

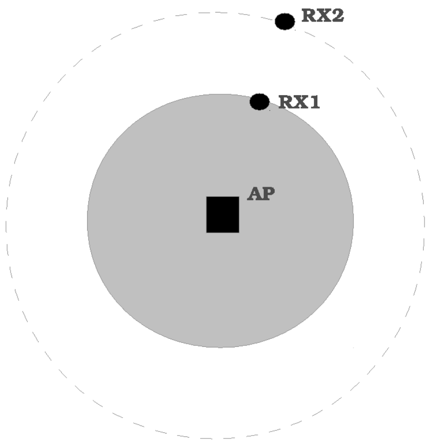

Figure 5.1: The TAS licensing mechanism restricts not only the radiated power but also the spectral density in a given geographical area. The grey area specifies the licensed regulation. For this case, RX2 will not be served since the transmit power will neglect the TAS agreement in the area.

difficulties; thus, leading to a more flexible management of the license since the receivers would be able to estimate the total amount of received power and send it back to the transmitters.

In addition, restricting the received power will not only respond to spectrum management considerations, but also to hardware ones. Indeed, the dynamic range of the Low Noise Amplifiers (LNA) directly impacts on their cost and should be designed carefully. The same happens with the A/D converter or to preclude saturation of the down-conversion chain of the receiver.

Coming back to regulatory aspects, and as commented in section I, received power constraints can also be presented as a power control mechanism for allowing the coexistence of different wireless services as the TAS licensing promotes. Oriented to a best use of the radio-spectrum, together with a continuously increasing demand of wide area communications, regulators might start to adopt the TAS licensing system [13]. First, since the spectrum usage is not only fragmented in frequency bands, but also in time slots and area spaces, a more efficient usage of the spectrum is promoted. Furthermore, this more restrictive license will have a lower price, opening the market to small and medium enterprises that would eventually provide wireless communication services. Nevertheless, the designers of these spectrum sharing networks will have to face the spatial spectrum restrictions that are intrinsic of this mechanism. Within a TAS, communications may work whenever their cumulated spectral density is not above a regulation threshold. Basically, the service will set its access point on the center of a microcell giving service to the users inside the cell, using proper scheduling and power control when attending several users at a time. At the same time, the service must guarantee that outside a circle around the AP, the maximum total signal power caused outside the TAS area is not above a given threshold, we denoted it as ρ. This threshold is in charge of the range control and can be, for instance, between 0 and 5 dB in order to attain a low or medium coverage, respectively.

Fig. 1 depicts this situation, where the access point is at the center of the circle. In order to establish communication with user 2 the base stations needs to transmit with a high power and, therefore, the amount of receive power in user 1 would violate the regulatory restrictions. When the AP has a single omnidirectional antenna, a proper power control is required to meet the constraint ρ. With multiple antenna at the AP the transmitted power can be combined together with a suitable beamformer.

Whenever the transmitter supports any power demand, the rate for a single user will be

$$R = \log_2\left(1 + \frac{\rho}{\sigma^2}\right) \quad \text{[bits/sec/Hz]} \tag{5.1}$$

where σ^2 is the noise power at the receiver. Clearly the power control will adapt the transmit power such that the global received signal is set to the regulation level.

A more complicated scenario is when more than one micro-cell is using the TAS. Furthermore, it is clear that the range of several APs overlap in a given area. This scenario is depicted in Fig. 2. Clearly, the possibility that three receivers, each corresponding to a different AP, stay on the overlapped area of the three ranges poses a more difficult problem to obey the mask, mainly because power control from each receiver works only for its corresponding access point, i.e. receivers are not coordinated. In addition, and for logistic reasons in services deployment, the APs are also not coordinated since they may be associated to a different vendor of communications services.

The following sections are devoted to solve this problem when the transmitters are equipped with multiple antennas and serve one user at a time. First, the optimal power control is derived. Later, the optimal transmit beamforming is formulated and a low complex decentralized solution is obtained.

5.2 System Model

We consider a scenario where K transmitters send information to their intended receivers with M antennas sharing frequency and time resources. We consider the natural case of $K = M$ which is common in cellular systems. Note also that usually, in practice, $K < 4$. The receivers have one antenna each. Recall that through this chapter, we denote $\mathbf{b}_k \in \mathbb{C}^{M\times 1}$ the transmit beamformer used by the k-th station

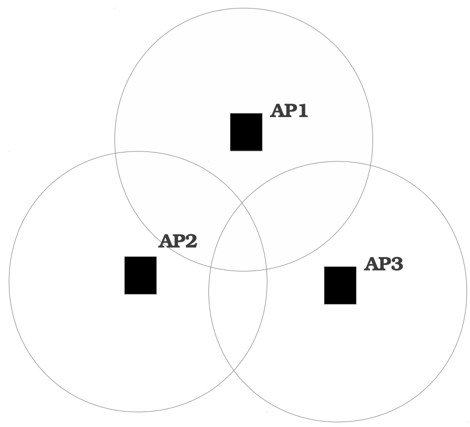

Figure 5.2: Difficulties arise when several base stations are located within a TAS area. The amount of created interference impacts on the total received power and limits the transmit power and, therefore, the range of the communication system.

which has unit norm. For notational convenience, we stack all the transmit beamformers in matrix $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_K]$. The available power or needed power by the k base station is p_k and we define the set of transmit powers by $\mathbf{p} = [p_1, \ldots, p_K]^T$. Moreover, the available power for the k-th base station is bounded by P^k_{max}, accordingly, and we define $\mathbf{P}_{max} = [P^1_{max}, \ldots, P^K_{max}]^T$

The link gain from the transmit beamformer i to the receiver j is

$$a_{ij} = \mathbf{b}_i^H \mathbf{R}_{ij} \mathbf{b}_i \tag{5.2}$$

where

$$\mathbf{R}_{ij} = \mathbf{h}_{ij} \mathbf{h}_{ij}^H \tag{5.3}$$

and $\mathbf{h}_{ij} \in \mathbb{C}^{M \times 1}$ is the spatial signature from the i-th base station to the j-th receiver. Matrix $\mathbf{A} \in \mathbb{R}^{K \times K}$ collapses all the link gains of the network $[\mathbf{A}]_{ij} = a_{ij}$.

As a novelty for TAS licenses, the system designer must take into account the amount of received signal power by all users which is restricted to ρ and it is assumed to be the same for all the standard receivers. For notational convenience we define $\boldsymbol{\rho}^T = [\rho \quad \ldots \quad \rho] = \rho \mathbf{1}$.

Note that our scenario is different from the broadcast channel: in our case the set of transmit beamformers and powers need to be calculated in a decentralized fashion since no cooperation between them is allowed. We target the solution of the following optimization problem

$$\begin{aligned} \underset{\mathbf{p},\mathbf{B}}{\text{maximize}} \quad & \sum_{k=1}^{K} r_k \\ \text{subject to} \quad & \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \\ & \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{max} \end{aligned} \tag{5.4}$$

where

$$r_k = \log_2\left(1 + \frac{a_{kk}p_k}{\sum_{j\neq k}^{K} a_{jk}p_j + \sigma^2}\right). \tag{5.5}$$

Considering that the objective of this chapter is provide an efficient solution of the already presented optimization problem, we will provide to approaches. First, we will consider that instead of the optimal sum rate, the telecom operator prefers to achieve certain values of QoS to their users while preserving the regulatory constraints. This perspective, lead to a centralized design and it is presented in the next section. Later, the study the decentralized study of the sum-rate optimization is targeted so that first the achievable rates are obtained and later the efficient solutions.

5.3 Centralized Design with both QoS and receive power constraints

5.3.1 Problem Formulation

The joint power and beamforming design for downlink systems under QoS restrictions problem has been studied in the past [5, 43]. Note that this problem can also embrace the interference channel when it is assumed a central unit which designs the beamforming and transmit power.

In those works, the approach approach is to minimize the total transmitted power (i.e. $\sum_{k=1}^{K} \|\mathbf{b}_k\|^2$) yet considering some SINR targets (γ_k) in order to ensure a QoS at the k-th receiver. In that centralized approach, the optimization problem then becomes

$$\begin{aligned} \underset{\{\mathbf{b}_k\}_{k=1}^{K}}{\text{minimize}} \quad & \sum_{k=1}^{K} \|\mathbf{b}_k\|^2 \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k \quad k = 1, \ldots, K \end{aligned} \tag{5.6}$$

which has extensively studied via the Perron-Forbenious theory and via semidefinite programming relaxation [5, 43]. In contrast to this method, the regulatory scenario imposes a total radiated power constraint. In that case, we need to impose a new constraint

$$\sum_{j=1}^{K} |\mathbf{b}_j^H \mathbf{h}_{jk}|^2 \leq \rho_k \quad k = 1, \ldots, K \tag{5.7}$$

where ρ_k is set by the regulatory scenario. With this, the primal problem (5.6) can be rewritten

$$\begin{aligned} \underset{\{\mathbf{b}_k\}_{k=1}^K}{\text{minimize}} \sum_{k=1}^{K} \|\mathbf{b}_k\|^2 & \\ \text{subject to} \qquad \text{SINR}_k \geq \gamma_k & \quad k = 1, \ldots, K \\ |\mathbf{b}_k|^2 \leq P_k & \quad k = 1, \ldots, K \\ \sum_{j=1}^{K} |\mathbf{b}_j^H \mathbf{h}_{jk}|^2 \leq \rho_k & \quad k = 1, \ldots, K \end{aligned} \tag{5.8}$$

where in this case the beamformers are not used by a single base station but a set of them.

The optimization problem (5.8) is a QCQP which is a non-convex problem. Therefore, obtaining the optimal beamformers $\mathbf{b}_k \quad k = 1, \ldots, K$ is cumbersome. Nevertheless, we can make use of the semidefinite relaxation [32] as follows

$$\begin{aligned} \underset{\{\mathbf{B}_k\}_{k=1}^K}{\text{minimize}} \quad & \sum_{k=1}^{K} \text{Tr}[\mathbf{B}_k] \\ \text{subject to} \quad & \text{Tr}[\mathbf{R}_{kk}\mathbf{B}_k] - \gamma_k \sum_{j \neq k}^{K} \text{Tr}[\mathbf{R}_{jk}\mathbf{B}_j] \geq \gamma_k \sigma_k^2 \quad k = 1, \ldots, K \\ & \sum_{j=1}^{K} \text{Tr}[\mathbf{R}_{jk}\mathbf{B}_j] \leq \rho_k \quad k = 1, \ldots, K \\ & \text{Tr}[\mathbf{B}_j] \leq P_j \quad j = 1, \ldots, K \\ & \mathbf{B}_k \succ 0 \quad \text{rank}[\mathbf{B}_k] = 1 \quad k = 1, \ldots, K \end{aligned} \tag{5.9}$$

.

where

$$\mathbf{B}_k = \mathbf{b}_k \mathbf{b}_k^H \quad \mathbf{R}_{jk} = \mathbf{h}_{jk} \mathbf{h}_{jk}^H. \tag{5.10}$$

The reformulation presented in (5.9) truly manifests the non-convexity of (5.8) which can be clearly noted by the rank-one constraint. If we drop this constraint, (5.9) becomes convex. Under this relaxation, we get a near-optimal solution of the problem but via gradient or interior point methods.

It is important to mention that once (5.9) is solved, we have to find the beamformer $\mathbf{b}_k$. In the following simulations we will use the eigenvector associated to the largest eigenvalue. Indeed, as it was shown in [22] as the number constraints is larger than 3 in the complex case, the resulting matrices $\{\mathbf{B}_k\}_{k=1}^{K}$ might have a rank larger than one. In that case, randomization methods can be used

5.3.2 Numerical results

We consider a three user scenario where the receivers are located at the intersection of the coverage of all base stations. For beamforming representation purpose, we assume line-of-sight channel model where the intended receiver is always at $\theta_0 = 0^{\text{o}}$ and the interfered users are at $\theta_{i1} = 10^{\text{o}}$ and $\theta_{i2} = -25^{\text{o}}$ and with fading -10 and -6 dB respectively.

In order to solve problem (5.9) without the rank constraint we used `CVX`, a package for specifying and solving convex programs [20, 21]. The noise variance is set to one and the maximum transmitted power is 10 for each user. Since we assume the same regulation body for all users, $\rho_k = \rho \quad k = 1,..,3$. Three different scenarios are simulated: one with no regulation constraints, $\rho = \infty$, and two different regulation constraints $\rho = 4, 5$. Moreover, the SINR targets are

$$\gamma_k = k \quad k = 1, ..., 3 \tag{5.11}$$

Figures 5.3-5 show the radiation patterns of the transmit beamformers. We will focus our study to user three which is seen from the users one and two at $\theta = 10^{\text{o}}$. When there is no regulation, the radiated interference to user three is high although the SINR target is achieved. In that situation the total received power by user three is 5.5963.

If we set the regulation constraints to $\rho = 5$, we can observe that both users 1 and 2 have reduced the radiated interference to user 3 in order to fulffill the mask (note that now the total received power by user three is 5). It is important to mention that both users 1 and 2 have increased the transmitted power in this scenario since the array gain at the intended receiver has decreased.

When we strength the regulatory constraint to be $\rho = 4$, the transmit beamformers from users 1 and 2 have totally rejected the interference to user 3 incurring in an array gain loss in the desired direction. Due to that the required transmitted power for these two users is very high. If we continue decreasing ρ the problem becomes infeasible.

For the sake of completeness, both the transmit and receive power are presented in table 5.1. Note that how the receive power fullfills the regulation tighter depending on the receiver. This is the case of receiver 3 whose QoS is higher and due to that its need in desired signal power strength fullfills with equality the regulatory constraints.

As a result of this numerical evaluation, it is clear that the regulatory constraints

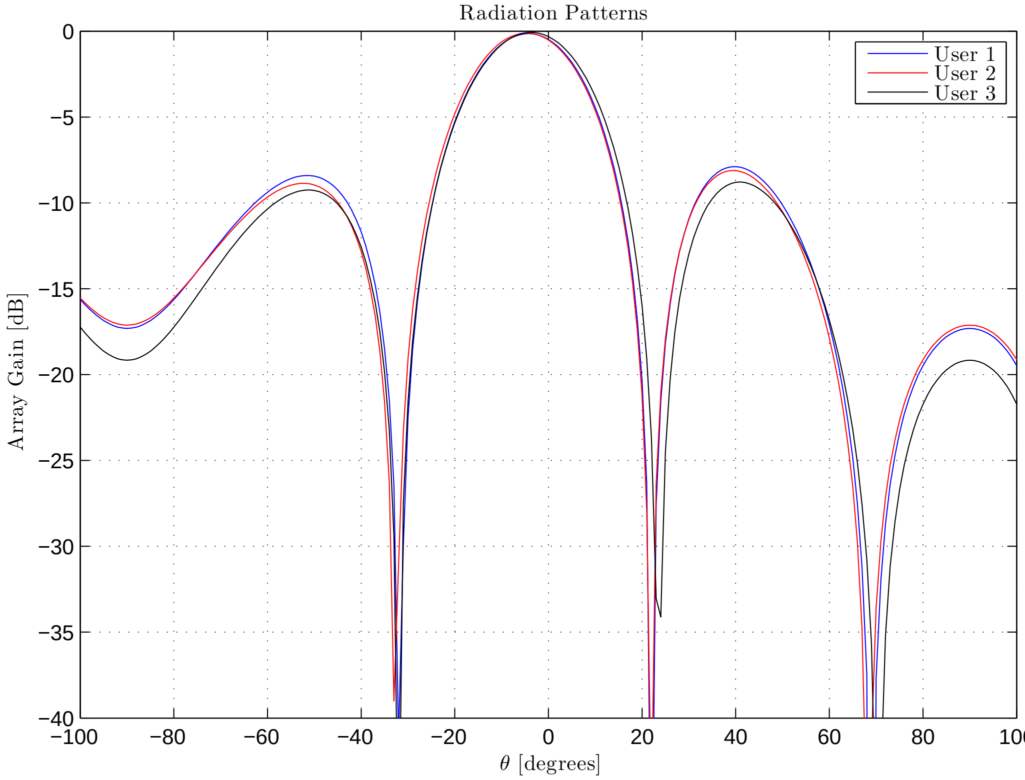

Figure 5.3: Radiation Patterns with no regulatory constraints

Table 5.1: Receive and Transmit Powers

Scenario	No regulation			$\rho = 5$			$\rho = 4$	
User	1	2	3	1	2	3	1	2
Transmitted Power [mW]	1.2072	3.3416	4.5107	1.4495	3.7492	4.1315	4.0049	8.2228
Received Power [mW]	2.1613	4.4523	5.5963	2.2062	4.2301	5.0000	2.4599	3.9025

modify the optimal designs and they play an important role in both the beamforming and power allocation design. As an extension to this studym in the next sections the decentralized and sum rate optimal design is analysed. Note that the difference with respect to the already presented work is remarkable and we will not longer focus on a set of QoS constraints but the overall network efficiency.

5.4 Characterization of the Achievable Rates

5.4.1 Rate and Power Pareto Region

We aim to find all optimal rate pairs of this communication system when the receivers implement single user detection and their received power is limited. Under this context, for a given set of transmit beamformers, **B**, achieving all optimal rate

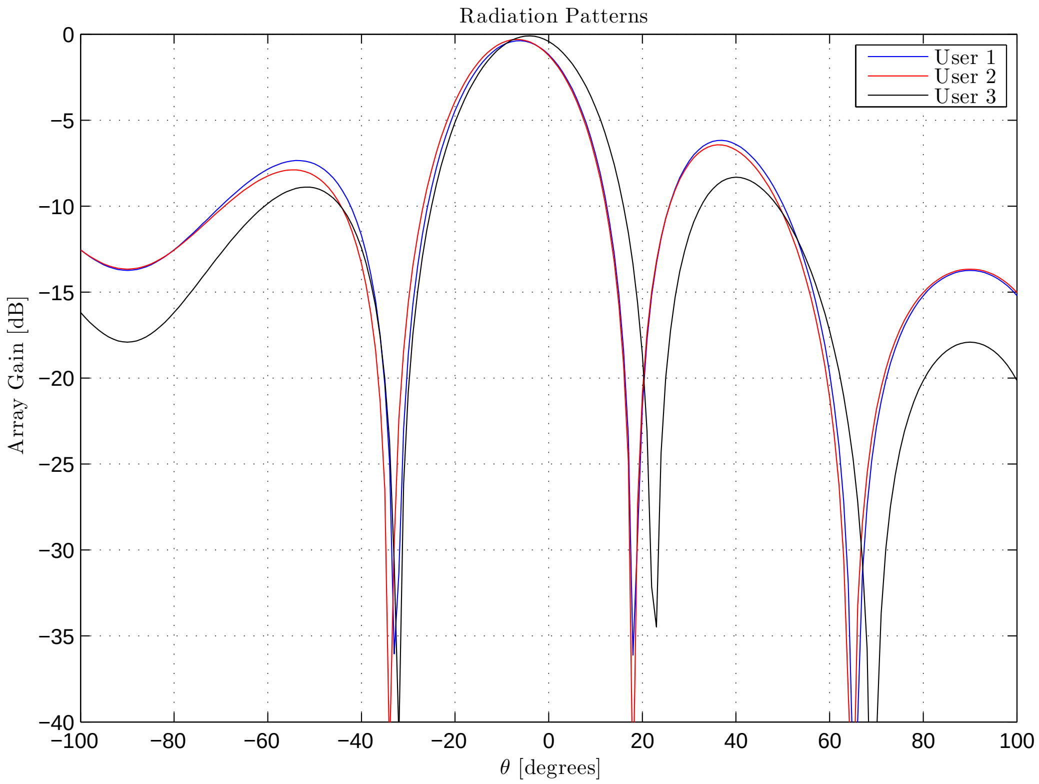

Figure 5.4: Radiation Patterns with $\rho = 5$

points is defined as the solution of the following multicriteria optimization problem:

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \mathbf{r} \\ \text{subject to} \quad & \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \\ & \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{max}. \end{aligned} \tag{5.12}$$

We can obtain an equivalent problem by operating the objective functions. We can change the vector objective function by $\mathbf{g} = [g_1, \ldots, g_K]$ with

$$g_k = \frac{\mathbf{a}_k^T \mathbf{p} + \sigma^2}{\mathbf{a}_k^{\star T} \mathbf{p} + \sigma^2} \tag{5.13}$$

where vector $\mathbf{a}_k$ is the k-th column of matrix $\mathbf{A}$ and $\mathbf{a}_k^\star$ is the same vector where in the k-th entry there is a 0 instead of a_{kk}.

Clearly, (5.13) is a linear fractional function. Thus, (5.12) is a MOLFP [26] which is an optimization problem that appears in different fields. Problem (5.12) can be transformed to a simpler problem with the help of the following theorem.

Theorem 5.1. $\mathbf{p}^*$ *is an optimal solution of* (5.12) *if and only if it is an optimal*

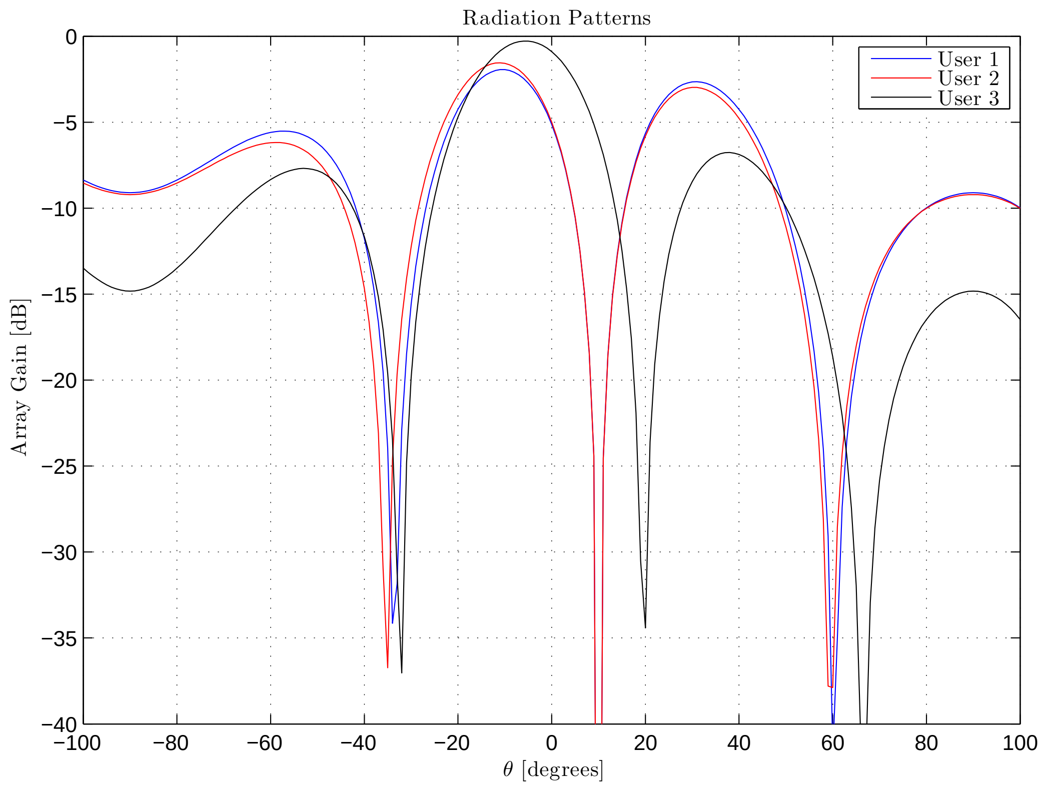

Figure 5.5: Radiation Patterns with $\rho = 4$

solution of

$$\begin{aligned} \underset{\mathbf{p}}{\textit{maximize}} \quad & \mathbf{h} \\ \textit{subject to} \quad & \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \\ & \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{max} \end{aligned} \tag{5.14}$$

where

$$[\mathbf{h}]_k = \mathbf{a}_k^T \mathbf{p} - \mathbf{a}_k^{\star T} \mathbf{p} = a_k p_k \tag{5.15}$$

Proof. The proof is mainly based on [47, Theorem 6.4.1]. We do not reproduce it here since it can be consulted in [47]. □

As the multiobjective function is independent of the scaling factor, the problem becomes

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \mathbf{p} \\ \text{subject to} \quad & \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \\ & \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{max}. \end{aligned} \tag{5.16}$$

Consequently, from theorem 5.1 we can establish that the maximum achievable rates are obtained when the transmitters work at the edge of the feasible power set since (5.16) actually obtains those points. This is a very important results since note that it is independent of the constraints. We will remark this fact in the next paragraphs.

Each component of the vector objective function is linear and the constraints are linear, therefore, the problem can be casted as a MOLP [15]. This optimization problem can be solved via the Multiobjective simplex method [55, 15], which is able to find the set of efficient solutions. Basically, the solving method relies on the weighted-sum method scalarization technique, which transforms the MOLP into

$$\begin{aligned} \underset{\mathbf{p}}{\text{maximize}} \quad & \mathbf{w}^T \mathbf{p}_k \\ \text{subject to} \quad & \mathbf{A}\mathbf{p} \preceq \boldsymbol{\rho} \\ & \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}_{max}. \end{aligned} \tag{5.17}$$

With this, for each vector $\mathbf{w}$ so that $w_i \in [0,1]_1^K$, and $\sum_{i=1}^K w_i = 1$, we obtain a rate Pareto optimal point. In figures 5.6 and 5.7 both the rate and the power achievable tuples are depicted for a scenario of two users, considering

$$\mathbf{A} = \begin{pmatrix} 1 & 0.2 \\ 0.2 & 1 \end{pmatrix} \tag{5.18}$$

The scenario parameters were

$$\mathbf{P}_{max}^T = [5 \quad 5] \quad \boldsymbol{\rho}_m^T = [3 \quad 3] \quad \sigma^2 = 1 \tag{5.19}$$

This result recasts and extends the result previously presented in [9], which was done for two users and with only transmit power constraint. We incorporate received power constraints and show that the achievable rate region boundary is attained at the power Pareto region, which is a function of the considered constraints. The next corollary remarks this fact.

Corollary 1: The achievable rate region of the interference channel is at the border of the power feasible set.

So far the rate region has been investigated: Now it remains open to determine which of those Pareto power points is desirable to be used and; in addition, how to properly construct matrix $\mathbf{A}$ which is determined by the beamformer designs.

5.4.2 Power Allocation

Obtaining the maximum sum-rate power allocation of an interference network is known to be very complex [1]. However, authors in [9] show that in noise limited scenarios (i.e. the amount of received interference is low w.r.t. the noise power level) and for the two-user case, the optimal power allocation strategy is that both transmitters transmit at the maximum available power. In other words, they work at the corner point of the power Pareto region, whenever this region is rectangular.

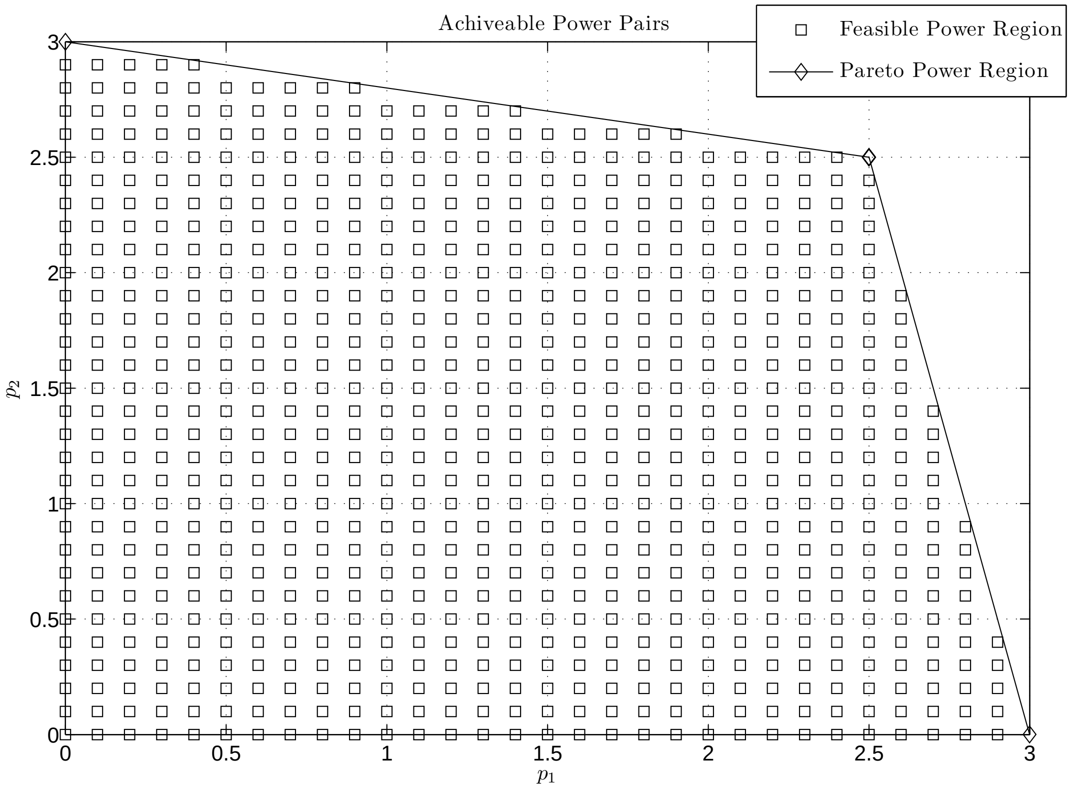

Figure 5.6: Power Pareto Region

The derivation is done via considering the convexity or concavity of the rate region.

The same derivation can be generalized for trapezoidal regions as the one in Figure 5.6. This is the case when a receive power mask is incorporated and the available power at each transmitter is above this mask, which is the desirable situation. However, for the K user case the analytical derivation is not so straight forward. The intuition says that the corner point in the power Pareto boundary that meets all the receive mask constraints with equality is a working point of interest.

Note that the power Pareto region and its corresponding rate region depend on the channel gains $\mathbf{A}$. In fact, whenever the transmitters have multiple antennas, it is reasonable they use them so that the generated interference is attenuated as much as possible; thus, fulfilling

$$\mathbf{A}\mathbf{p}^* = \boldsymbol{\rho} \tag{5.20}$$

Therefore, our target is to design the system so that it works at the equilibrium which is (5.20). Let us now consider the feasibility of (5.20) (i.e. when it exists a positive solution of $\mathbf{p}^*$).

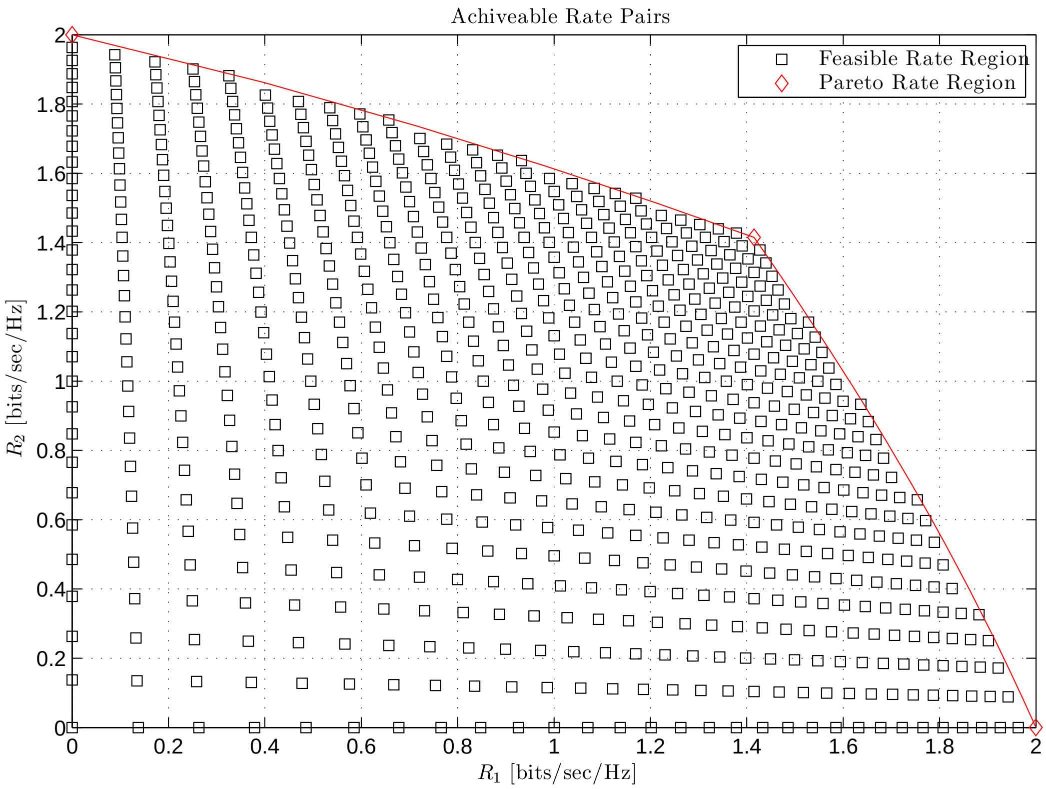

Figure 5.7: Rate Pareto Region

Theorem 5.2. *A positive solution of* (5.20) *exists if*

$$1 > \sum_{j \neq k}^{K} \frac{a_{jk}}{a_{jj}} \quad k = 1, \ldots, K \tag{5.21}$$

Proof. In [24] is derived that, considering a positive matrix $C \in \mathbb{R}^{K \times K}$ and a vector $\mathbf{b} \in \mathbb{R}^{K \times 1}$ so that

$$b_i \gg 0 \quad i = 1, \ldots, K \tag{5.22}$$

then if all $i, i = 1, \ldots, K$

$$b_i > \sum_{j \neq i}^{K} c_{ij} \frac{b_j}{c_{jj}} \tag{5.23}$$

then $\mathbf{C}$ is invertible and $\mathbf{C}^{-1}\mathbf{b} \gg \mathbf{0}$. Particularized for our case, since we have assumed that all users have the same regulatory constraint, we obtain (5.21). $\square$

Note that (5.21) has to guide the beamforming design so that the final beamforming scheme is able to fulfil (5.21). Before getting into the beamformer design in Section V, we propose a decentralized design for the transmit powers, such that, given the channel gains, the receive mask constraints in (5.20) are met.

5.4.3 Working point and decentralized Power Control

Whenever the transmitters have multiple antennas, it is possible to design beamformers that properly attenuate the interference that is generated towards the unintended receivers; thus, fulfilling (5.21). For this reason, the system that is proposed in this chapter consists of twofold: beamforming design and power control. Next section is devoted to the decentralized beamforming design. Now we focus on the decentralized power control, which aims to attain the MURC as desirable working point.

Under the premise that each receiver can communicate feedback only to its transmitter, it is clear that the possibilities reduce to a mere gain control. In other words, when receiver k experiences a received power above/below the mask ρ a feedback is produced to its corresponding transmitter in order to reduce/increase to some amount its transmitted power. For the sake of completeness we summarize next the iterative and distributed power control that we propose in [40].

An adaptive design of the feedback response from the transmitter can be an LMS-like rule such as

$$p_k(n) = \alpha p_k(n-1) + \beta\left(\rho - \rho_k(n-1)\right), \tag{5.24}$$

where $p_k(n)$ is the available power of transmitter k at iteration n, and $\rho_k(n-1)$ is the power experienced by receiver k after the $n-1$ iteration. Note the decentralized character of (5.24), since a selected transmitter is only able to handle the sensed mask only from its own receivers (i.e. subscribed users).

Some choices are available for α and β. The first algorithm is motivated by [16], but focusing on the SINR targets. If we focus on the regulation constraints instead, the differential dynamic is

$$\frac{d}{dt}\rho_k = \gamma\left(\rho - \rho_k(t)\right) \tag{5.25}$$

where the total received power at terminal k, ρ_k, is driven towards the desired mask level ρ. In order to implement this equation only with local measurements, we assume that the k-th transmitter strives to evolve p_k as if the interference contribution to the received power was not going to change. The equation for this dynamic is

$$\frac{d}{dt}p_k(t) = \frac{\gamma}{a_{kk}}\left(\rho - \rho_k(t)\right). \tag{5.26}$$

The convergence of this rule is controlled by parameter γ and the eigenvalues of the matrix $\mathbf{A}$ defined previously [40]. Under this context, the difference equation becomes

$$p_k(n) = p_k(n-1) + \frac{\gamma}{a_{kj}}\left(\rho - \rho_k(n)\right) \tag{5.27}$$

for $j = 1, \ldots, K$ (i.e. the transmitter updates its transmit power whenever any of the receivers has an excess of receive power). It is important to mention that with this policy, any user belonging to cell k that enters in another cell, will ask to reduce the power assigned to it, even arriving to zero and therefore losing the link. The reason is that the mask is full of interference, coming from the new cell it entered.

As a matter of fact, this method is devoted to fulfill the received power constraint that corresponds to the working point of (5.20). This point is of interest only when the interference level is low, as (5.21) dictates. Next section presents a transmit beamforming design for obtaining a recommended low interference power level and also sum-rate optimization.

5.5 Decentralized Transmit Beamforming with Received Power Constraints

Whereas so far in the previous chapter our aim was to obtain an efficient transmit beamforming design for interference networks, now we put our attention to the regulatory scenario where the receive power constraints play an important role as it was described in the centralized design section.

In fact, it is expected that ρ must appear somehow in the $\lambda_{kj,k}^{j\,K}$ parameters assignment since the array gain design must take into account the receive power restrictions. For the sake of completeness we again present the decentralized transmit beamforming designs in order to consider them as a starting point for our proposal which is presented in the last subsection.

5.5.1 Existing Transmit Beamformers

As we observed in the previous chapter, the optimal transmit beamforming design in a multiantenna interference channel depends on the desired and interference signal power levels with respect to the noise level as we have shown in the last chapter. When the SNR is low the optimal design for the two user case is the matched filter

$$\mathbf{b}_k^{\mathrm{MF}} = \frac{\mathbf{h}_{kk}}{\|\mathbf{h}_{kk}\|}, \tag{5.28}$$

whereas when SNR is very high, zero-forcing beamformer is the best option

$$\mathbf{b}_k^{\mathrm{ZF}} = \frac{\left(\mathbf{I} - \mathbf{R}_k^I\right)\mathbf{h}_{kk}}{\|\left(\mathbf{I} - \mathbf{R}_k^I\right)\mathbf{h}_{kk}\|}, \tag{5.29}$$

where $\mathbf{R}_k^I$ is the matrix that contains in its columns the interference channel vector of transmitter k (i.e. $\{\mathbf{h}_{jk}\}_{j=1,j\neq k}^{K}$). These results where obtained in [29].

There are also two more designs, namely the virtual-SINR beamformer [58]

$$\mathbf{b}_k^{VS} = \frac{\left(\sum_{j=1, j\neq k} \gamma_k \mathbf{R}_{jk} + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_k}{\| \left(\sum_{j=1, j\neq k} \gamma_k \mathbf{R}_{jk} + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_k \|}, \tag{5.30}$$

which presents an intermediate behaviour between the MB and the ZFB. Note that $\gamma_k, \quad k = 1, \ldots, K$ are degrees of freedom that are not easy to design. The most used scheme is when $\gamma_k = P_{max}$ as the so-called MMSE transmit beamformer results [42]. However, other values can be used as for instance it was done in [59] in the context of multicell communications.

The other option is the EIG beamformer

$$\left(p_k \mathbf{R}_{kk} + \sigma^2 \mathbf{I}\right) \mathbf{b}_k^{EIG} = \lambda_{\max} \left(\sum_{j=1, j\neq k} p_k \mathbf{R}_{jk} + \sigma^2 \mathbf{I} \right) \mathbf{b}_k^{EIG}, \tag{5.31}$$

which can outperform the VB in some cases as we observed in the previous chapter. Note that EIG dates back to 2G (i.e. second generation mobile communications), when it was used as the first beamformer at reception that incorporated the receiver AGC constraints [27].

5.5.2 Proposed Transmit Beamformer

None of the previous beamformers at transmission take into account in their design the received power constraint, which is our case of interest. Considering these constraints and assuming that mask ρ is fulfilled, the rate delivered to each user is given by

$$R_k = \log_2 \left(\frac{\sigma^2 + \rho}{\sigma^2 + I_k} \right) \quad \text{[bits/sec/Hz]}, \tag{5.32}$$

where I_k is the total amount of interference by user k. It is clear that (5.32) assumes that the transmitter is able to achieve the maximum power level at all the receivers/users in the scenario as we pointed out in the previous section. Under such circumstances the optimum policy to maximize the sum-rate is ZF, i.e. to null out the interference in the denominator of (5.32). The sum rate for this case is

$$R_{sum} = K \log_2 \left(1 + \frac{\rho}{\sigma^2} \right) \quad \text{[bits/sec/Hz]}. \tag{5.33}$$

Nevertheless, zero forcing implies that when the channel of desired and the channel of interference are similar, i.e. risk/intersection zone of Fig. 5.2, the transmit power requirements could be enormous in order to fulfill the constraint implicit in (5.32). In other words, the power used by user k named p_k^* would be far above the available power. In consequence, we have to add an additional constraint on the available power for each transmitter, P_{max}. That is

$$p_k^* \leq P_{max}, \tag{5.34}$$

where unit norm beamformers are considered and $P_{max}^k = P_{max} \quad k = 1, \ldots, K$. More important, as we show next we have to abandon the idea of beamforming independent of the available power settings as the zero forcer is. The rest of this section is devoted to design a beamformer that fulfills both received and available power constraints.

Assuming that there exist a power tuple that fulfills with equality the regulatory constraints (i.e. (5.20) holds), we can express the available power as

$$p_k^* = \frac{\rho \sum_{q=1}^{K} A_{kq}}{\Delta}, \tag{5.35}$$

where Δ is the determinant of matrix $\mathbf{A}$ and A_{kq} is the cofactor of element a_{kq}. By inserting (5.35) into (5.34), the available power constraint is transformed into

$$\frac{\rho}{P_{max}} \sum_{q=1} A_{kq} \leq \Delta = \sum_{q} a_{kq} A_{kq}. \tag{5.36}$$

It is worth taking into account that the cofactor of the link gain matrix entries that are outside the main diagonal use to be negative, i.e.

$$A_{kq} \leq 0 \quad k \neq q. \tag{5.37}$$

This is a realistic assumption since it is expected that after beamforming the channel gain towards the desired receiver is bigger than the gains towards the unintended receivers. In summary, the transmit power constraint can be reformulated as

$$a_{kk} A_{kk} \geq \frac{\rho}{P_{max}} \left(\sum_{q=1}^{K} A_{kq} \right) + \sum_{q \neq k} a_{kq} |A_{kq}|. \tag{5.38}$$

The major advantage of formulating the problem in terms of the cofactors of the elements of $\mathbf{A}$ is that they help to concentrate all that is not known in a local or decentralized design. The available power constraint in (5.38) can be written as

$$a_{kk} \geq \frac{\rho}{P_{max}} \gamma_0 + \sum_{q \neq k}^{K} \beta_{kq} a_{kq} \tag{5.39}$$

where

$$\gamma_0 = \frac{\sum_{q=1}^{K} A_{kq}}{A_{kk}} \tag{5.40}$$

and

$$\beta_{kq} = \frac{|A_{kq}|}{A_{kk}} \quad k \neq q. \tag{5.41}$$

Note that

$$\gamma_0 + \sum_{q \neq k}^{K} \beta_{kq} = 1. \tag{5.42}$$

It is clear that by abandoning the goal of having the zero forcing solution, which is optimum without the constraint on the available power at the transmitter, we are going to support a finite SIR at the receiver. Therefore, we consider SIR_k in the design, which will turn out to be related with the feasibility condition (5.21). The SIR at the MURC (i.e. when (5.20) is fulfilled) is

$$\mathrm{SIR}_k = \frac{a_{kk} p_k^*}{\rho - a_{kk} p_k^*}. \tag{5.43}$$

Therefore,

$$a_{kk} p_k^* = \left(\frac{SIR_k}{1 + SIR_k} \right) \rho. \tag{5.44}$$

By inserting (5.35) into (5.44) we obtain

$$a_{kk} = \left(\frac{SIR_k}{1 + SIR_k} \right) \frac{A_{kk}}{\sum_{q=1}^{K} A_{kq}} \left(a_{kk} - \sum_{q \neq k}^{K} \beta_{qk} a_{qk} \right). \tag{5.45}$$

This expression is used in order to incorporate the SIR into the available power constraint (5.39). The resulting inequality is

$$\left(\frac{SIR_k}{1 + SIR_k} \right) \frac{1}{\gamma_0} \left(a_{kk} - \sum_{q \neq k}^{K} \beta_{qk} a_{qk} \right) \geq \|\mathbf{b}_{kk}\| \frac{\rho}{P_{max}} \gamma_0 + \sum_{q \neq k}^{K} \beta_{kq} a_{kq}. \tag{5.46}$$

Under this context, the transmit beamformer design that fulfills the inequality (5.46) is

$$\left(\mathbf{R}_{kk} - \sum_{j=1, j \neq k} \mathbf{R'}_{jk} \right) \mathbf{b}_k = \lambda_{\max} \left(\sum_{j=1, j \neq k} \mathbf{R'}_{jk} + \frac{\rho \gamma_0}{P_{max}} \mathbf{I} \right) \mathbf{b}_k, \tag{5.47}$$

where $\lambda_{\max}$ is the corresponding maximum generalized eigenvalue that should meet the following condition

$$\lambda_{\max} \geq \left(\frac{1 + SIR_k}{SIR_k} \right) \gamma_0. \tag{5.48}$$

Note also that if $\lambda_{\max}$ fulfills (5.48) then it is positive and guarantees semidefinite positiveness, that is

$$a_{kk} > \sum_{q\neq k}^{K} \beta_{qk} a_{qk}, \tag{5.49}$$

which is similar condition that the one in (5.21). Nevertheless, we will extensively simulate this scenario so that it is ensured that when the beamformers are properly designed, (5.21) is always fullfilled and; therefore, there is always a positive power solution.

Finally, beamformers in (5.47) can be reformulated as

$$\left(\mathbf{R}_{kk} + \frac{\rho\gamma_0}{P_{max}}\mathbf{I}\right)\mathbf{b}_k = (\lambda_{\max}+1)\left(\sum_{j=1, j\neq k}\mathbf{R'}_{jk} + \frac{\rho\gamma_0}{P_{max}}\mathbf{I}\right)\mathbf{b}_k, \tag{5.50}$$

which can be considered as the decentralized EIG [51] that incorporates TAS constraints. The major claim concerning EIG beamforming in TAS scenarios is that, holding transmit power constraints, it achieves minimum degradation with respect to the performance of zero forcing, which is sum-rate optimal in unbounded transmit power scenario. The numerical simulation section supports this statement with numerical evaluations. Next sub-section comments on the design of γ_0 and β_{kq} in order to attain the desired behaviour of (5.50). Recall that this procedure was mentioned in chapter 3 where the multicriteria array gain problem was considered.

5.5.3 Parameter settings

As the design is for uncoordinated transmitters, the resulting beamfomer in (5.50) should not depend on the other link gains. Without any a priori knowledge a practical approach is:

- to assume a symmetric scenario and, therefore, $\beta_{kq} = \beta_0$. Then from (5.42) we obtain

$$\gamma_0 + \beta_0 (K-1) = 1; \tag{5.51}$$

- to design the parameter γ_0 so that the beamformers tend to ZFs whenever the scenario requires it; that is when P_{max} and/or the number of transmit antenna are high. From (5.50) a ZF results if $\gamma_0 = 0$; therefore, γ_0 has to tend to zero with the increase of P_{max} or the number of antenna, M. The proposed design is $\gamma_0 = \frac{1}{MP_{max}+C}$, where C is a constant, whose setting is explained in the simulation section. Note that that the closest is γ_0 to zero, the higher is the SIR and also the easier is to fulfill (5.48).

We comment that by inspection of (5.40) and (5.41) for the zero-forcing case (i.e. when $A_{kq} = 0$) it results $\gamma_0 = 1$ and $\beta_{kq} = 0$, respectively. These values do not agree with the practical design that we propose. However, by substituting $\gamma_0 = 1$ and $\beta_{kq} = 0$ in (5.39) it results $a_{kk} \geq \frac{\rho}{P_{max}}$, which is fulfilled by a MF and not a ZF as initially assumed; thus, resulting a contradiction.

The answer to this paradox is that the selfish MF does not optimize sum-rate due to the interference that each transmitter creates towards the other co-existing communications. Therefore, the transmitters would react by decreasing $|A_{kq}|$ in order to deviate from the MF and obtain a more altruistic design that tries to zero-force the interference with their available transmit power. This intuitive reasoning brings us to a game between ZF and MF; however, obtaining the optimal design in a centralized way is complex as the original problem stated in (5.4) is non-convex and coupled.

However, our goal is to solve (5.4) in a decentralized fashion and the practical setting that we propose for γ_0 and β_{kq} close to 0 and $1/(K-1)$, respectively, help us to obtain a valid design from the EIG beamformer of (5.50) as next section shows.

5.6 Numerical Examples

The system we propose consists in two steps:

- First the transmit beamformers are designed such that they do not require knowledge of full link gain matrix $\mathbf{A}$, but only of those channel gains where each beamformer participates (i.e. each receiver should broadcast to the network its channels with each of the transmitters).

- Second, the power control is carried out to attain the MURC point (we recall that in the power design, the obtained beamformers play a key role in the link gain as formulated in (5.20) and (5.2)) and, as a consequence, in the feasibility of the MURC point.

In the first subsection we evaluate the beamformers. In order to compare them we plot the achieved rate regions under the received power constraint mask. In the second sub-section, we evaluate the whole system (i.e. jointly beamformer and power control) by computing the achieved ergodic sum rate.

5.6.1 Rate Regions

The first simulation scenario consists of two base stations and two receivers. The following numerical results have been obtained considering that transmitters are equipped with two antenna and the channels are randomly generated with a Rayleigh distribution. Noise variance is set to one and all the points are obtained via a Monte Carlo simulation of 1000 realizations.

We call our proposal in (5.50) EIG and compare it with the ZF and with the equivalent Virtual-SINR or MMSE beamformer of the form

$$\mathbf{b}_k^{VS} = \frac{\left(\sum_{j=1,j\neq k}\mathbf{R}_{jk} + \frac{1}{P_{max}}\mathbf{I}\right)^{-1}\mathbf{h}_k}{\|\left(\sum_{j=1,j\neq k}\mathbf{R}_{jk} + \frac{1}{P_{max}}\mathbf{I}\right)^{-1}\mathbf{h}_k\|}. \tag{5.52}$$

Note that the comparison for this case is fair since this beamformer does not depend on the transmit power, but on the available one, P_{max}, and; therefore, can be distributively calculated as EIG beamforming does.

In $\gamma_0 = \frac{1}{MP_{max}+C}$ we set $C = \rho$. In this way, for low values of MP_{max} with respect to ρ the beamformer that results from (5.50) presents the same loading as the so-called MMSE transmit beamforming in (5.52).

We first consider short range TAS (i.e. low available transmit power P_{max}). Two scenarios (Fig. 5.8 and 5.9) are obtained by varying the available power $P_{max} = 1, 3mW$ (i.e. 0 and 4.77 dBm, respectively) and maintaining the regulatory constraint to $\rho = 1mW$ (4.77 dBm).

For each plot, for each channel realization the three beamformers are computed and the link gain matrix is obtained. From this matrix a power Pareto region and corresponding rate Pareto region are obtained (i.e. as in the example of Fig. 5.7). For the sake of clarity we do not plot the power Pareto regions that results from considering the received power constraints after applying each of these beamformers (i.e. (5.16) with (5.2)). However, we should have them present to justify some of the results.

Starting with $P_{max} = 1mW$, note that in this case an standing alone transmitter will not fullfill the regulation mask, which in praxis is an anomalous case of low interest. The resulting rate region is shown in Fig. 5.8. We note that all the beamformers achieve maximum sum-rate at the MURC of the rate region (i.e. -1 slope point). It is observed that the highest sum-rate is obtained with the EIG beamforming.

In the next case, where $P_{max} = 3mW$, the available power is able to fullfill the regulation mask even if one transmitter is standing alone in the scenario. For this case Fig. 5.9 shows that EIG beamforming still outperforms both Virtual SINR and zero-forcing beamformer. We comment that, as it is expected, if the available power P_{max} is further increased all three beamformers would collapse to the squared ZF rate region and achieve the maximum sum-rate value that is established by (5.33) (i.e. 1 bit per user and channel use for ρ equal to 1mW).

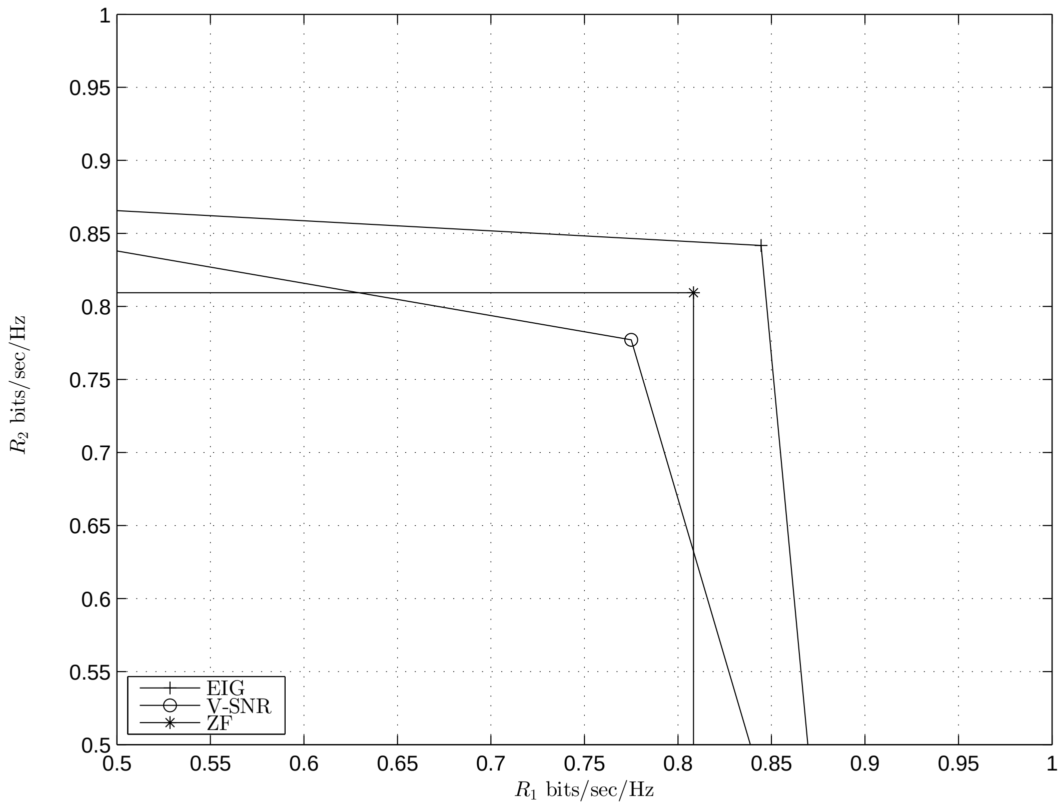

Figure 5.8: Rate region when $\rho = 1mW$ and $P_{max} = 1mW$. +: EIG, o: VS and *: ZF.

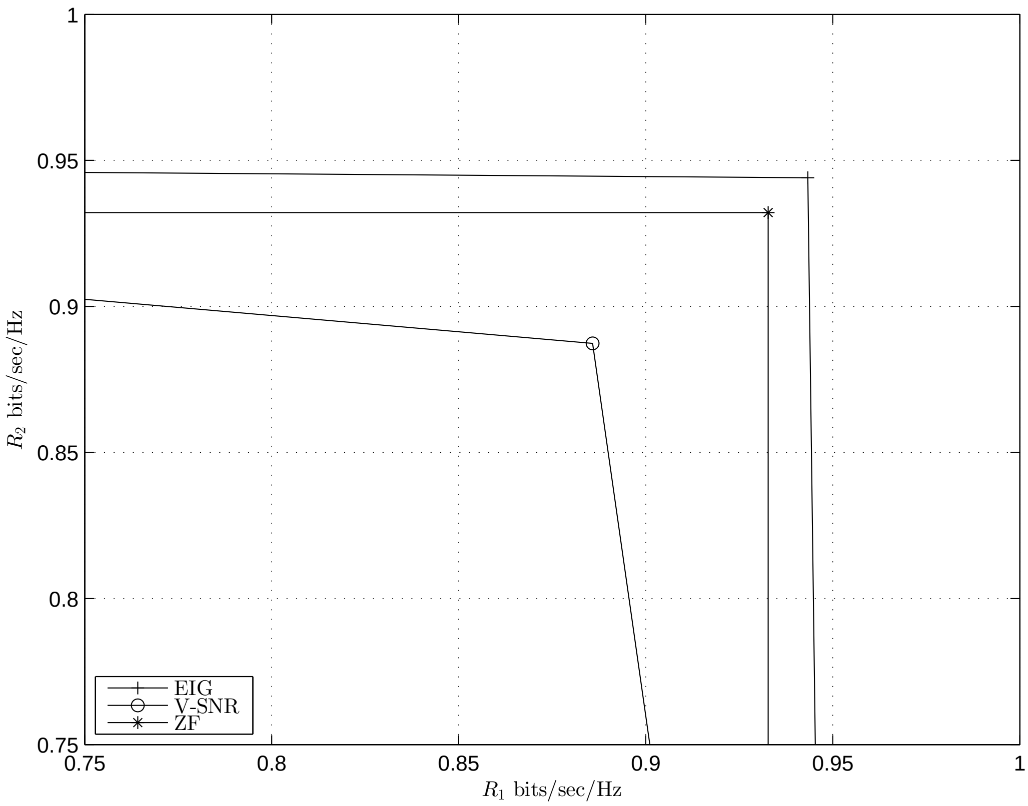

Figure 5.9: Rate region when $\rho = 1mW$ and $P_{max} = 3mW$. +: EIG, o: VS and *: ZF.

5.6.2 Sum-rate Analysis

Once the beamformers are designed, each transmitter reaches the desired transmitting power via the iterative and distributed mechanism that is formulated in (5.27). In this subsection we evaluate the ergodic sum-rate that is obtained in the two and also three user case. The channel realizations are generated as the previous case. Now 2000 Monte Carlo runs are considered.

We assume that the regulatory constraint, ρ, is fixed and we vary the available transmit power, P_{max} from -3 dB to 3 dB over a fix ρ. For negative values we have observed in the simulations that the maximum sum-rate solution may correspond to a vertex of the power Pareto region that is either on the x- or y-axis. In praxis this solution is not of interest for the system due to its unfairness. Also, in general, on-off signalling requires more bandwidth for the same rate and transmit power than other systems.

However, the incurred observed loses in sum-rate are negligible when the MURC vertex is considered instead. In this way, all communication pairs are on. Therefore, sum-rate has been computed at the MURC vertex, with the power control that we proposed before.

In the scenario with 2 transmitters, Fig. 5.10 shows the better behaviour of EIG in the mid-range regime, where P_{max} is not either too high or low with respect to ρ. Note that for $\frac{P_{max}}{\rho} = 1$ (e.g. 0 dB)the attained sum-rate is the same as the one attained at the MURC of Fig. 5.8; this is due to having used the same parameters in both points of these two figures.

Fig. 5.11 plots the outage or percentage of realizations that each beamformer fails the feasibility condition (5.21) and illustrates the robustness of EIG in (5.50) in front of the VS beamformer of (5.52).

Fig. 5.12 and 5.13 show that the sum-rate that is attained by each of the 3 beamformers tend to be similar as either ρ or the number of antenna increases, respectively. As the number of antenna per transmitter increases, each transmitter has better interference rejection capabilities while maintaining a medium array gain to the intended receiver, that is why the range where EIG is the best solution is shortened. Note, however, that EIG keeps on presenting the best performance. The maximum sum-rate for this scenario can be computed with (5.33); in Fig. 5.10 is equal to 2 bits/sec/Hz (i.e. $2\log_2(1+1)$).

In Fig. 5.12 $\rho = 5mW$ and therefore the maximum sum-rate is equal to 5.17 bits/sec/Hz (i.e. $2\log_2(1+5)$). We recall that these bounds render exact values when P_{max} is high enough so that ZF can be implemented as if there were no constraints on the available power at transmission.

Fig. 5.14 and 5.15 show the sum-rate when there are 3 transmitters that are

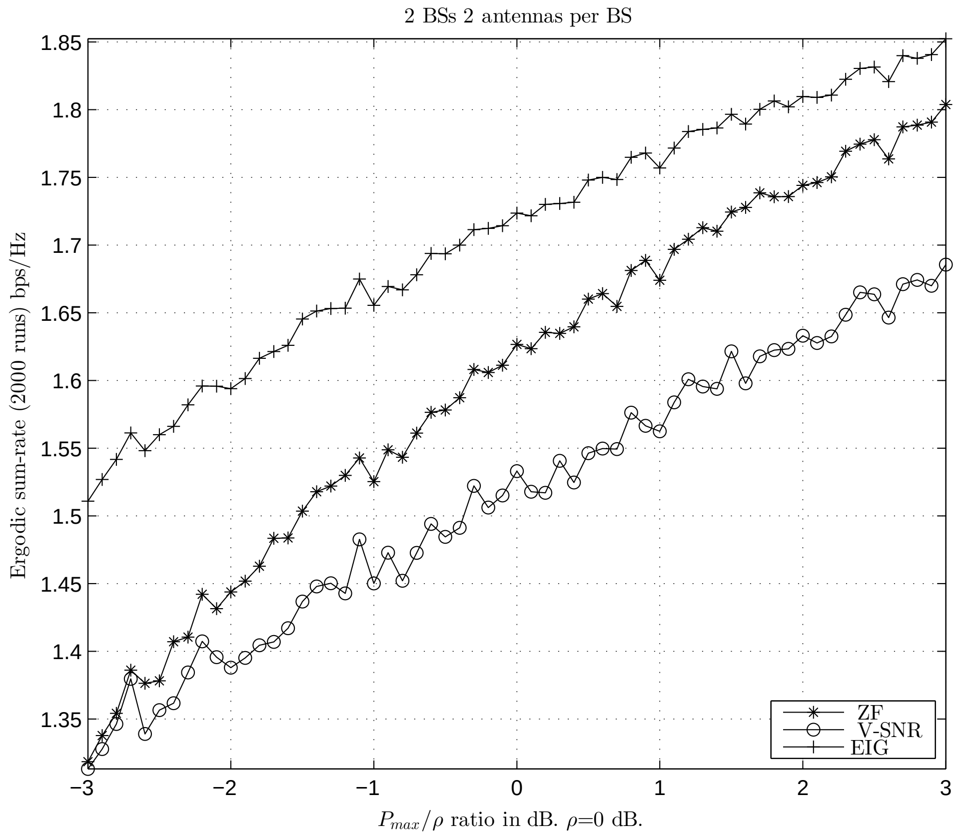

Figure 5.10: Sum-rate of a 2-user scenario where the transmitters are equipped with 2 antenna, $\rho = 1mW$.

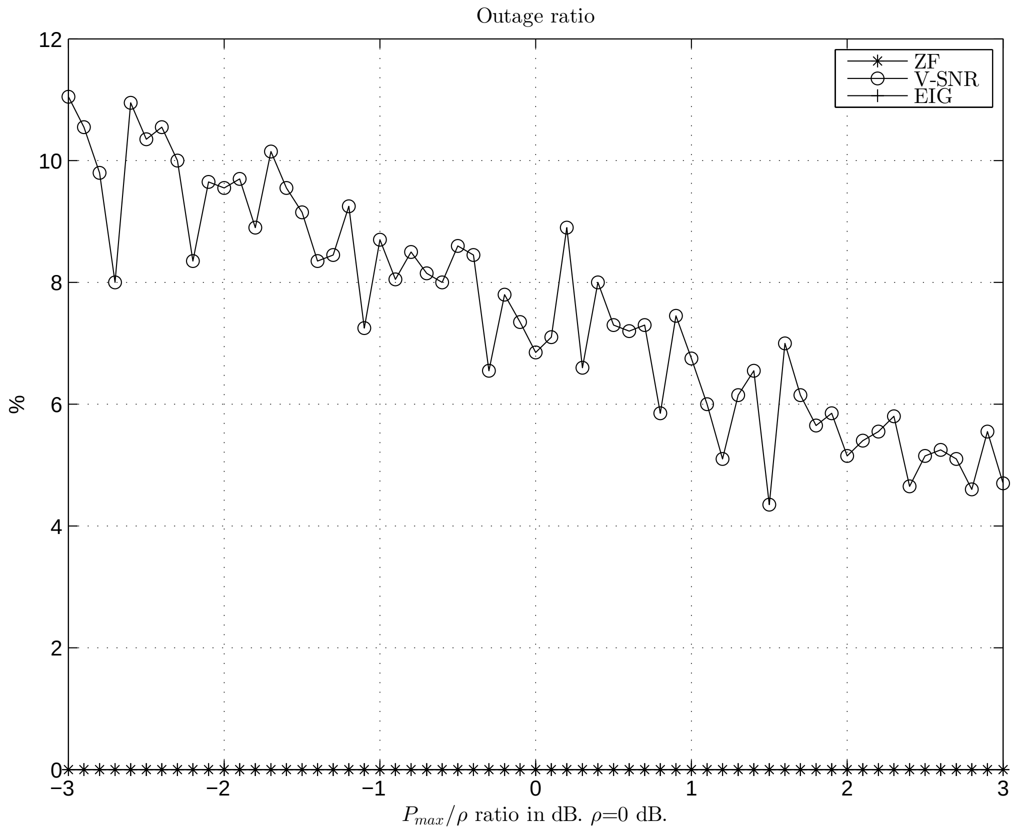

Figure 5.11: Outage for a 2-user scenario where the transmitters are equipped with 2 antenna, $\rho = 1mW$.

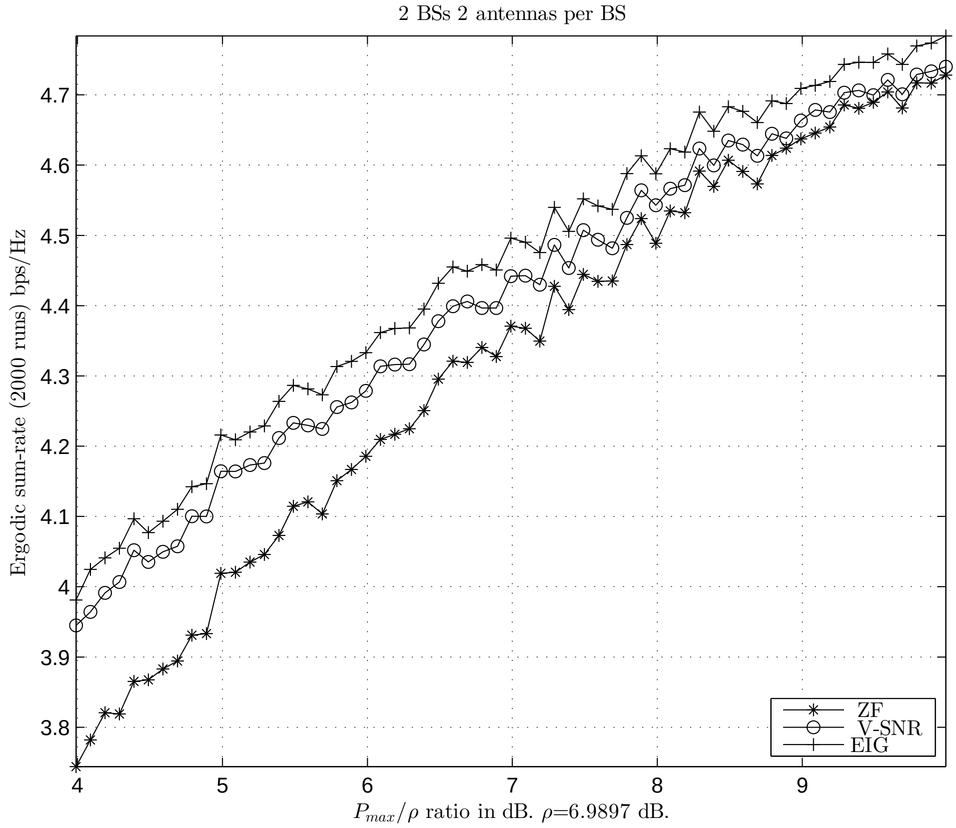

Figure 5.12: Sum-rate of a 2-user scenario where the transmitters are equipped with 2 antenna, $\rho = 5mW$.

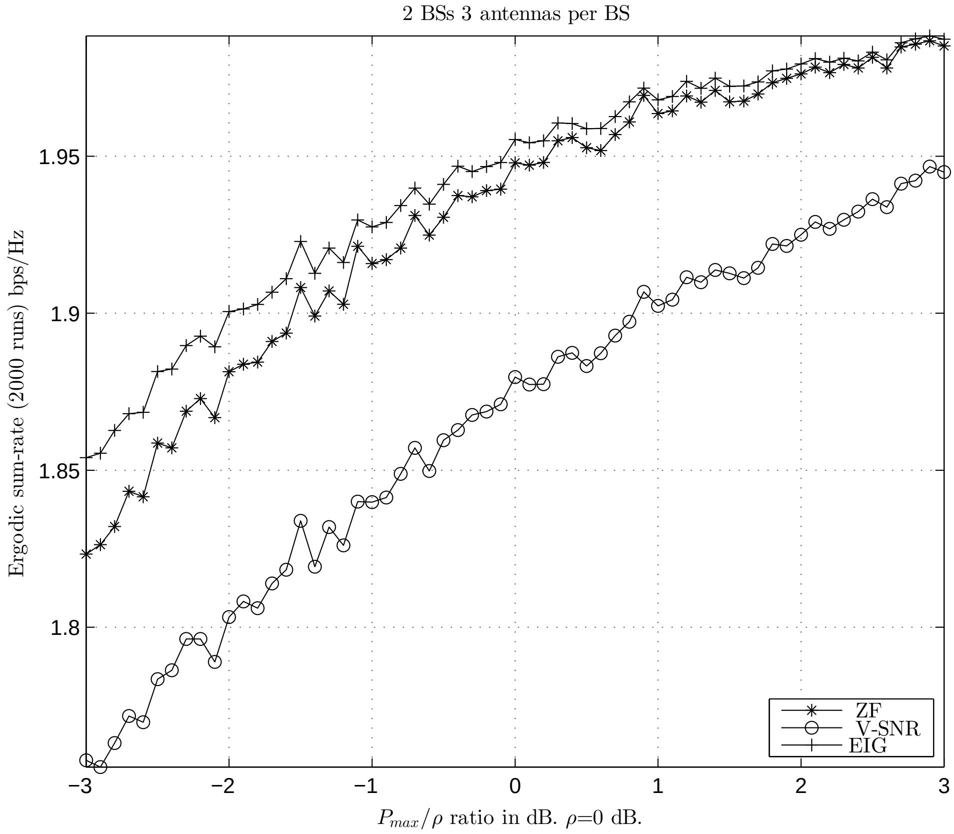

Figure 5.13: Sum-rate of a 2-user scenario where the transmitters are equipped with 3 antenna, $\rho = 1mW$.

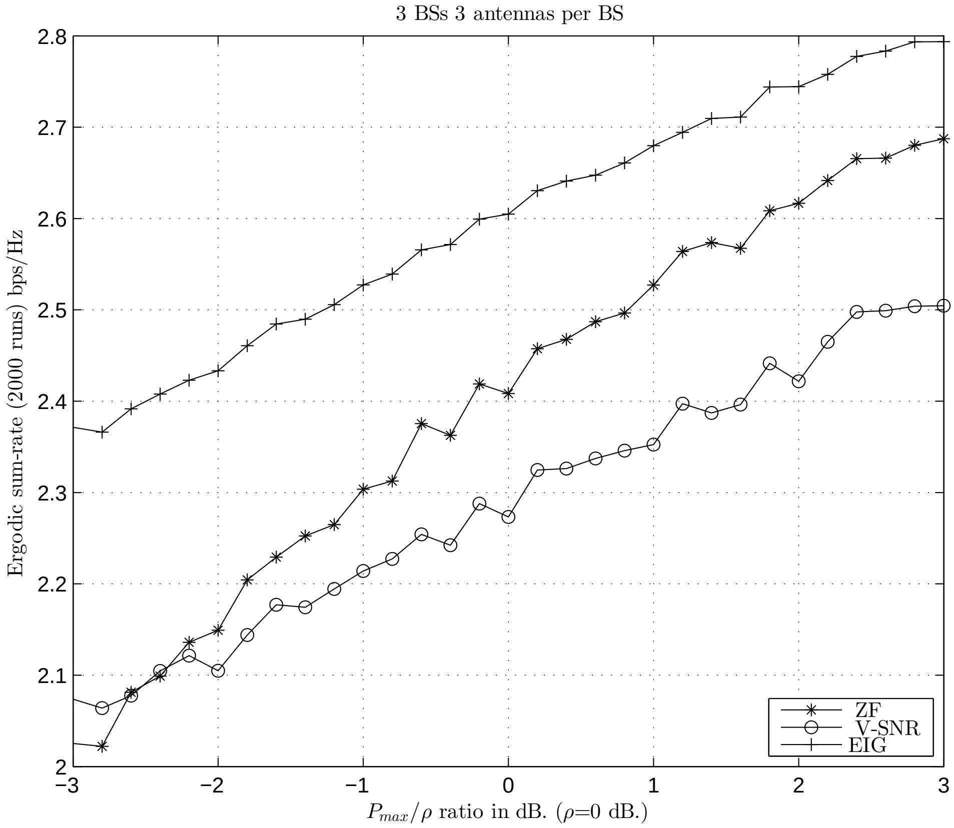

Figure 5.14: Sum-rate of a 3-user scenario where the transmitters are equipped with 3 antenna, $\rho = 1mW$.

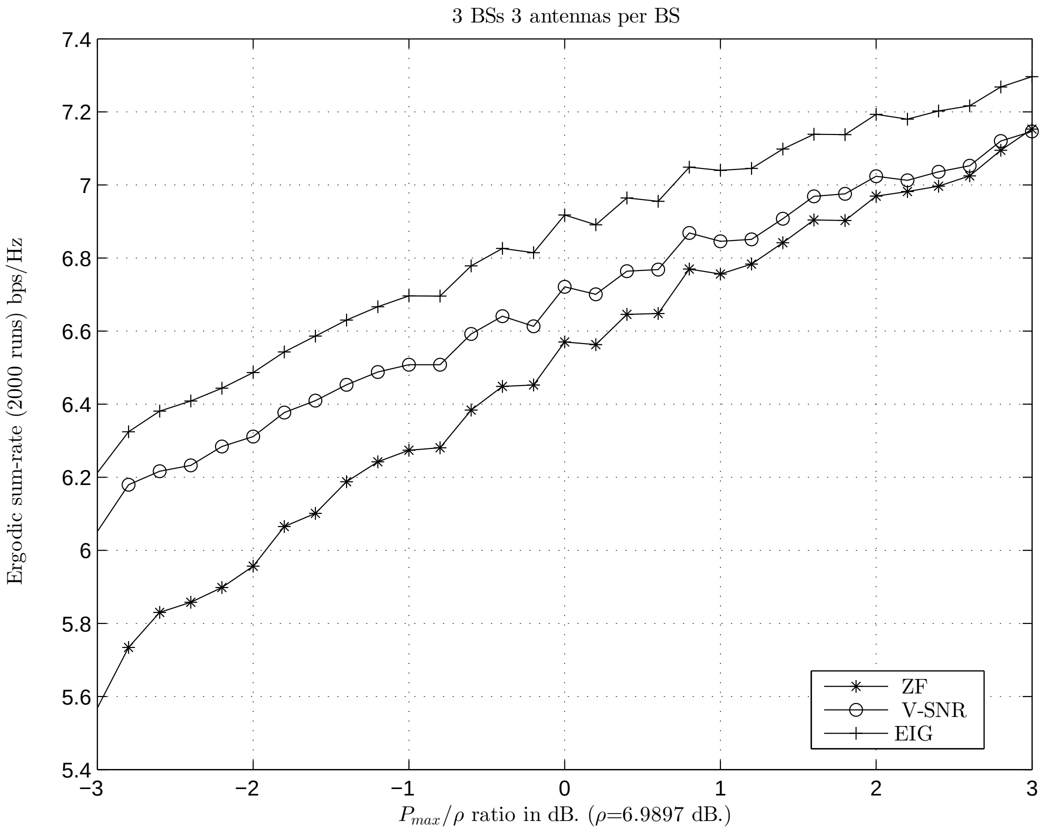

Figure 5.15: Sum-rate of a 3-user scenario where the transmitters are equipped with 3 antenna, $\rho = 5mW$.

equipped with 3 antenna each. In Fig. 5.14 $\rho = 1mW$ and the maximum sum-rate for this scenario equal to 3 bits/sec/Hz (i.e. $3\log_2(1+1)$). In Fig. 5.15 $\rho = 5mW$, with maximum sum-rate equal to 7.75 bits/sec/Hz (i.e. $3\log_2(1+5)$). Note that we can draw the same conclusions than in the two-user scenario: when P_{max} is high with respect to ρ all beamformers tend to the ZF.

5.7 Summary

We have considered the constraint on received power as the best mechanism for both: i) regulating radiated power in order to manage interference and improve the coexistence of different spectrum-sharing networks; ii) avoiding misfunctioning of receivers' RF and digital initial stages. TAS licensing is the underlying spectrum management system that frames the present work, which confines users within different coverage areas. We first study the centralized case where each receiver requires an specific QoS apart from to fullfill the regulatory restrictions. Later, when considering the decentralized and sum rate optimal case, we provide the optimal power control policy via considering the problem as a multicriteria optimization problem. An specific working point of the power Pareto region is chosen and a decentralized power control is proposed. Next, the optimal transmit beamforming scheme that corresponds to the constrained received power policy is revisited and it results to be nonconvex and coupled. In order to cope with this problem, we take a practical point of view that results in a decentralized beamforming method with a closed-form implementation. Numerical results show the good performance of the proposed decentralized beamformer and power control in front of existing ones.

Chapter 6

Conclusions and Future Work

This thesis has analysed the optimal beamforming and power allocation designs for spectrum sharing networks and also it has provided suboptimal and low computationally, decentralized algorithms. Moreover, the paradigm of interference networks with receive power constraints as a regulatory mechanism has been investigated and a decentralized beamforming and power allocation has been provided.

Next sections provide an overview of the presented works as well as future research lines that has been identified during the realization of the investigations.

Achievable Rates of the K-user MISO Interference Channel

The Pareto rate region of the MISO interference channel has been studied and characterized by means of relaying on multicriteria optimization. From an academical point of view, the contribution has an important impact since it provides an unified framework for describing the existing works. From a more practical point of view, we observe that any transmit beamforming design is indeed, the computation of K-1 parameters, corresponding to the array gains to both intended and nonintended receivers.

The single multicriteria optimization method can be solved via different scalarization techniques that offer different features so that the designer is able to chose any of them depending on his/her preferences. Some further investigations are described in the following:

- Other communication models can be eventually analysed from the multicriteria optimization perspective so that the achievable rates can be easily determined. This the case of [8] where single beam MIMO systems are studied. Furthermore, in [38] it is used for describing the trade-off between power transfer and security constraints and; thus, other general communication scenarios can be also evaluated.

- Computing the beamforming parameters can be done either in a centralized or decentralized fashion. This latter is preferable as the communication overhead is reduced. Thus, distributed parameter computations must envisaged and; in addition, there might also take into account the variability of the channel leading to adaptive beamforming mechanism for spectrum sharing systems.

Transmit Beamforming for the MISO Interference Channel

EIG beamforming was presented as novel transmit beamforming that generalizes the optimization in the MISO interference channel. Indeed, the signal power levels assumptions from EIG beamforming is derived are the more relaxed ones. On the other hand, its design is justified in terms of the reciprocal version of a receiving beamforming design with AGC constraints and the novel notion of antenna directivity for multiusers communications.

With this, as EIG beamforming outperforms the current designs is wide range of situations and transmit power, its use is adequate for next generation wireless systems due to also its low complexity. Other topics regarding this design can be also addressed:

- The extension of the EIG beamforming for the MIMO precoding matrix is not straightforward and it must be carefully studied. Nevertheless, the extension of the EIG beamforming structure to the MIMO case might improve the performance of the current designs.

- It remains open to design the transmit beamforming when a single symbol is sent to various users simultaneously leading to a multicast interference channel. In that case, the system performance is determined by the user with lowest achievable rate and; thus, the optimization problem becomes more difficult.

Power Control and Transmit Beamforming for the MISO Interference Channel with Receive Power Constraints

Clearly, when the coverage of spectrum sharing techniques is increased, higher transmit powers are needed and a more carefully power allocation and regulation is needed. Indeed, that was our objective when presenting the TAS licensing system which apart from opening the doors to SME to the wireless business, it allows a mechanism for increasing the spectral efficiency in adjacent networks. As a matter of fact, limiting the receive power is a procedure to restricting the radiated power in a given area so that neighbouring areas might support a more aggressive frequency reuse.

The optimal design of power control and beamforming was studied but; unfortunately, the complexity of these optimal designs make them impossible to implement them in real systems. In order to solve this problem, we provide a low complex beamforming design that, jointly with an efficient power control, is able to maintain the receive power level under a certain threshold. As an extension to the presented work, we identified the following:

- It remains open how for instance IEEE 802.11 Wi-Fi protocol can be modified in order to integrate the novel regulatory spectral restrictions. The use of our proposal under this protocol might be advantageous and system level performance metrics should be done.

- Interference alignment mechanism jointly with receive power constraints is an interesting study since this technique is one of the most promising for MIMO interference networks. In that case, a central unit is assumed and the full (or partial) channel state information is available in a central unit. Both power allocation and precoding design can be studied and promising results are expected.

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